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Bifurcation Analysis of an Elastic Ring Supported by Springs

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ABSTRACT

Until now, we have shown the structure of bifurcation of the problem of the inextensible elastic ring through the numerical results[1]. As the next step of our research, in this paper, the problem of determining possible equilibrium states of buckled extensible elastic ring subjected to a uniform pressure is discussed. Especially, the ring is supported by the geometrically nonlinear or linear sprigs (shown in Fig.1). This problem is known as the bifurcation problem in which there exist many bifurcation solutions and limit points depending on the nonlinearity of the strain and the characteristic of spring.

The governing equations for ring are derived from the nonlinear strain-displacement equation of elasticity. The nonlinear strain-displacement equation in terms of othogonal curvilinear coordinate system is given as follow:

$$\epsilon = \frac{dv}{ds} - \frac{w}{R} + \frac{1}{2}(\frac{dv}{ds} - \frac{w}{R})^2 - z(\frac{d^2w}{ds^2} + \frac{1}{R}\frac{dv}{ds}) \qquad (1)$$

where v is the tangential displadement, w is the radial displacement, R is the initial radius of the ring and z is the distance from the mid line. The nonlinear stiffnes equations depending on the pressure parameter and the tangential stiffness matrix are derived by using finite element procedure based on the principal of stationary total potentail energy.

In order to find the bifurcation solution at a bifurcation point, the bifurcation technique in conjunction with the arc-length method is applied effectively. To obtain the solutions on fundamental path, arc-length method which is known as one of the incremental solution procedure is used. Especially, it is easy to track the solution beyond limit point. At a bifurcation point, Hosono's method[2] to obtain the bifurcation solution is used. The applicability of the method has been already shown in bifurcation problems[3,4]. This method is a powerful method to obtain the increment solution on the singular point as a limit point or a pichfork bifurcation point. Moreover, we need not calculate the second-order increment equation which is used by the static perturbation method , as well as eigen value problem.

In our numerical results for the pressure it is shown that there exist the bifurcation solutions.

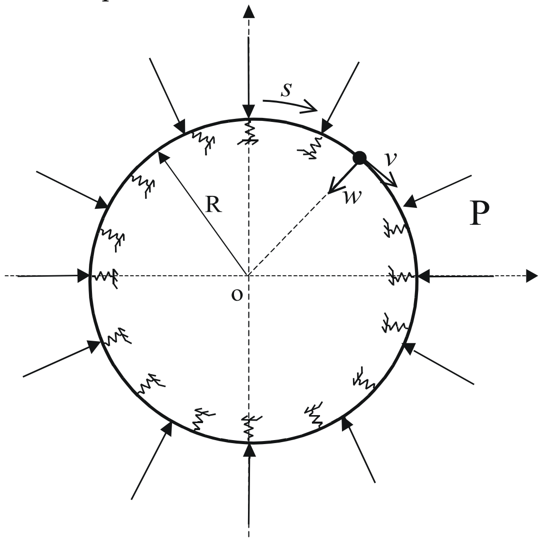

Fig. 1 Problem of an extensible elastic ring supported by springs and its coordinate

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