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# REHABILITATION OF THE LOWEST-ORDER RAVIART-THOMAS ELEMENT ON QUADRILATERAL GRIDS

*** Pavel Bochev[1] and Denis Ridzal[2]**

[1] Sandia National Laboratories, MS1320,
P.O. Box 5800, Albuquerque, NM 87185,
USA
pbboche@sandia.gov

[2] Sandia National Laboratories, MS1318
P.O. Box 5800, Albuquerque, NM 87185,
USA
dridzal@sandia.gov

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## ABSTRACT

A recent study [1] points out that the convergence of finite element methods that use $H(\Omega, \mathrm{div})$-compatible finite element spaces deteriorates on non-affine quadrilateral grids. This deterioration is particularly troublesome for the lowest-order Raviart-Thomas element, because it implies loss of convergence in some norms for finite element solutions of mixed and least-squares methods. In this paper we show that a reformulation of finite element methods in terms of the so-called natural mimetic divergence operator [2] restores the order of convergence.

Reformulations of mixed Galerkin and least-squares methods for the Darcy equation illustrate our approach. We prove that reformulated methods converge optimally with respect to a norm involving the mimetic divergence operator. Furthermore, we prove that standard and reformulated versions of the mixed Galerkin method actually *coincide*, but that the two versions of the least-squares method are genuinely different. The surprising conclusion is that the degradation of convergence in the mixed method on non-affine quadrilateral grids is superficial, and that the lowest order Raviart-Thomas elements are safe to use in this method. However, the breakdown in the least-squares method is real, and there one should use our proposed reformulation.

## REFERENCES

[1] D. N. Arnold, D. Boffi, and R. S. Falk, “Approximation by quadrilateral finite elements”, *Math. Comp.*, Vol. **71**, 909–922, 2002.

[2] M. Shashkov, *Conservative finite difference methods on general grids*, CRC Press, Boca Raton, FL, 1996.