8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

Dynamic Modeling and Simulation of Multi-Tethered Satellite Formations in Halo Orbits

Jun Zhao and Zhiqin Cai *
Department of Engineering Mechanics, Dalian University of Technology,
2 Linggong Road, Ganjingzi District, Dalian, Liaoning, 116024, P. R. China
E-mail: zhqcai@dlut.edu.cn

Key Words: *Hill's problem, Halo orbits, multi-tethered satellite formations, dynamic simulations*

Introduction

In the last few years, a number of space-based observatory missions have sparked great interest in multi-spacecraft formation flight in a periodic orbit (Halo). However, these formation flights are even more challenging with their own unique problems of stability, formation reconfiguration and many other associated issues. Therefore, the propulsive conducting tethers and spin-stabilized tether systems have been proposed in place of on-board propulsion systems to form and maintain satellite formations [1].

Some relevant studies have been carried out. Recently, Wong and Mirsa [2] represented a preliminary study of a multi-tethered system in CR3BP and focus mainly on the cases when the tethers are very close to the horizontal plane, but the spinning tethered system in Halo orbits was not considered.

In this paper , taking nonlinear gravitational terms and the rotation of the Sun-Earth frame into account in the Hill's problem[3], we model and simulate the three-dimensional dynamics of the spinning multi-tethered satellite systems arranged in a hub-spoke formation in the periodic Halo orbits near the L2 libration point.

Derivation of the Equations of Motion

The equations of motion of the system are derived using the method of Lagrangian in the body-centered coordinates (E-XYZ) in the Hill's problem and then transformed to the coordinates with its origin at the second libration point(L2-xyz). The nondimensional equations of motion for the parent satellite can be expressed as

$$\ddot{x}-2\dot{y}-3(1+x)+\frac{(1+x)}{R^3}+\sum_{i=1}^{N}F_{xi}=\frac{Q_X}{m_t n^2 l_0 \gamma_L}$$
$$\ddot{y}+2\dot{x}+\frac{y}{R^3}+\sum_{i=1}^{N}F_{xi}=\frac{Q_Y}{m_t n^2 l_0 \gamma_L},\ \ddot{z}+z+\frac{z}{R^3}+\sum_{i=1}^{N}F_{zi}=\frac{Q_Z}{m_t n^2 l_0 \gamma_L} \quad (1)$$

Where $l_0=\left(\frac{\mu_e}{n^2}\right)^{\frac{1}{3}},\gamma_L=\left(\frac{1}{3}\right)^{\frac{1}{3}},m_t=m_p+\sum_{i=1}^{N}m_i,$

$$\frac{1}{R^3}=\frac{1}{\gamma_L^3}\left(1+2x+x^2+y^2+z^2\right)^{-\frac{3}{2}}$$

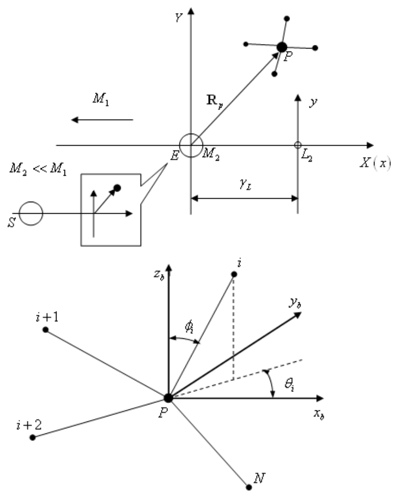

Figure 1 Geometry of the multi-tethered satellite formation in Hill's problem

μ_e represents the gravitational parameter of Earth , n is the rotation rate of the Earth about the Sun, F_{xi} , F_{yi} and F_{zi} are the tether influence terms and Q_{xp} , Q_{yp} and Q_{zp} are the generalized forces. The equations governing the pitch and roll rotation of the i^{th} tether are

$$\begin{aligned} &\hat{\hat{l}}_i^2 \sin^2\phi_i \ddot{\theta}_i + \left(\cos\theta_i \ddot{y} - \sin\theta_i \ddot{x}\right) \hat{\hat{l}}_i \sin\phi_i + \hat{\hat{l}}_i^2 \dot{\phi}_i \left(1+\dot{\theta}_i\right)\sin 2\phi_i + \hat{\hat{l}}_i \sin\phi_i \left[\left(2\dot{y}+x+1\right)\sin\theta_i + \left(2\dot{x}-y\right)\cos\theta_i\right] = \frac{Q_{\theta_i}}{m_i n^2 l_0^2 \gamma_L^2} \\ &\hat{\hat{l}}_i^2\ddot{\phi}_i + \left(\cos\theta_i\ddot{x}+\sin\theta_i\ddot{y}\right)\hat{\hat{l}}_i\cos\phi_i - \ddot{z}\hat{\hat{l}}_i\cos\phi_i - \frac{1}{2}\hat{\hat{l}}_i^2\left(1+\dot{\theta}_i\right)^2\sin 2\phi_i + \hat{\hat{l}}_i\cos\phi_i\left[\left(2\dot{x} - y\right)\sin\theta_i - \left(2\dot{y} + x+1\right)\cos\theta_i\right] = \frac{Q_{\phi_i}}{m_i n^2 l_0^2\gamma_L^2} \end{aligned} \quad (2)$$

Numerical Simulations

The above dynamics of a three-body tethered satellite formation is simulated numerically using Gear’s method for the stiff nonlinear ordinary differential equations in MATLAB. For space-saving, it can only be given in figure 2 (a) and (b) that the parent satellite is placed in the Halo orbit with nondimensional initial values $x(0)=-0.198127302428$, $z(0)=0.1795891228957$, $\dot{y}(0)=1.0882212923128$, $y(0)=\dot{x}(0)=\dot{z}(0)=0$

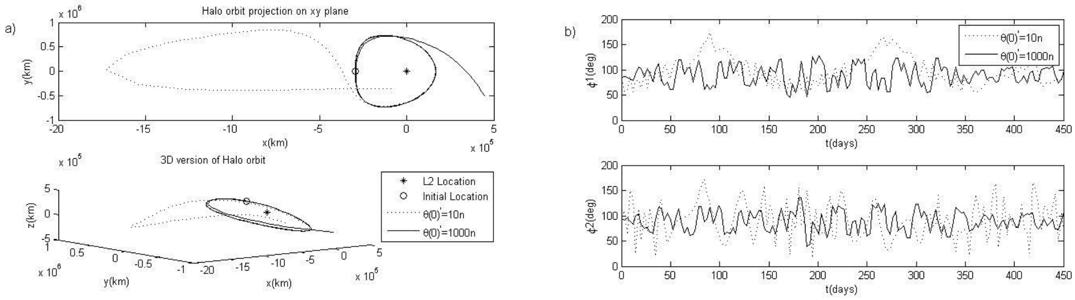

Figure 2 Halo Orbit Motion and Out-of-Plane Tether Librations
$l_1 = l_2 = 100km, \theta_1(0) = 5\deg, \theta_2(0) = 85\deg, \phi_1(0) = 85\deg, \phi_2(0) = 95\deg$

Figure 2 (a) shows that the stability of the parent satellite and (b) shows out-of-plane librations of sub-satellites with two different in-plane initial tether angular velocities in two periods of the Halo orbit(T=178.85days).It can be seen that a key factor in the design of a spining multi-tethered satellite formation system is the choice of the spin rate which can be used to provide centrifugal stiffening to the tethers and to stabilize the motions of the parent satellite over the long term.

Conclusion

It is concluded that it is necessary to have both an appropriate initial spin rate and to keep the configuration stable by the active control for the multi-tethered satellite formation systems in Halo orbits.

Acknowledgment

This work is supported by National Natural Science Foundation of China(10472017).

References

[1]Johnson, L., Gilchrist, B., Estes, R. D., and Lorenzini, E., “Overview of Future NASA Tether Applications,” NASA Technical Report 1998-0237034, 1998.

[2] Wong, B. Misra, A.K., “Dynamics of Lagrangian Point Multi-tethered Satellite Systems”, Journal of the Astronautical Sciences, Vol. 53, No. 3, p 221-250, (2005).

[3] B.F.Villac, “Dynamics in the Hill problem with Applications to spacecraft maneuvers”, Ph.D. Dissertation, The University of Michigan, USA, 2003