8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

A MORE RELIABLE FINITE ELEMENT PROCEDURE VIA CONTROLING THE CONDITION NUMBER OF THE SYSTEM

***Sina Mohajeri[1], Pouyan Broumand[2], Amir R.Khoei[3], Mostafa Zeinali[4], Leila Farrokhpour[5]**

[1] MS Student of Civil Engineering, BS Student of Computer Engineering
Sharif University of Technology, Tehran, Iran
sina@sinamohajeri.com, http://www.sinamohajeri.com
[2] MS Student of Civil Engineering
Sharif University of Technology, Tehran, Iran
p.broumand@gmail.com
[3] Associate Prof. Center of Excellence in Structural and Earthquake Engineering
Dep. of Civil Eng., Sharif University of Technology, P.O. Box. 11365-9313, Tehran, Iran
arkhoei@sharif.edu
[4] BS Student of Civil Engineering
Sharif University of Technology, Tehran, Iran
mostafa.zeinali@gmail.com
[5] BS Student of Civil Engineering
Sharif University of Technology, Tehran, Iran
leila.farrokhpour@gmail.com

Key Words: *Finite Element Method, Condition Number, Reliability, accuracy.*

ABSTRACT

Through the analysis of a mechanical system, the better numerical model is obviously the one which produces more reliable results in a cost-efficient manner. When using Finite Element method as the modelling strategy several parameters affect the accuracy and reliability of the results. These include the type and size of the generated mesh, type and order of the used elements, accuracy of the input data, solution procedure details and etc. But all these parameters are qualitative while only a quantitative criterion can be helpful to evaluate the level of accuracy and reliability.

Finite Element method and all its variations in both linear and nonlinear schemes require the solution of a system of linear equations (In the case of a nonlinear problem, it is finally divided into several linear iterations) for which, the condition number of the system is a suitable indicator of the reliability of the computed results. In fact all the effects of the mentioned parameters can be found in the total stiffness matrix of the system which identifies the condition number of the linear system of equations. This shows that the condition number can be a reasonable explanatory for accuracy of the results in a finite element problem.

In nearly all scientific computing, calculations with real numbers are performed using floating-point arithmetic. On most scientific computers, floating-point numbers may be represented in single or double precision. The important factor in these representations

is the *unit round-off* ($0.5\times2^{1-t}$) which is around $0.5\times2^{1-24}\simeq0.6\times10^{-7}$ for single and $0.5\times2^{1-54}\simeq0.56\times10^{-16}$ for double precision. It can be shown that in solving linear systems of equations, *unit round-off* is approximately proportional to $\kappa(A)\times u$ in which $\kappa(A)$ is the condition number of the multiplier matrix and u is the *unit round-off* of the machine. So with an increase in the condition number of a system of linear equations (the condensed total stiffness matrix of the system in FE method), there will be a noticeable decrease in the reliability of the results of the system.

As it can be seen in different ordinary numerical problems the condition number of FE stiffness matrices is around 10^4-10^8 and even higher in more complex systems, which indicates that this phenomenon should be taken into account while choosing the type and number of the elements to model the problem. Besides, it is seen that this number usually increases with the decrease in the mesh size and with the application of higher order and complex unstructured elements, which can make the results completely unreliable.

One solution to this problem can be the application of more than double precision for real number representations, which can decrease the efficiency of the calculations and may not be suitable. The other solution can be the use of a method to make the procedure control the condition of the system and decrease the negative effect of the described problem.

In this method one specific type of element is chosen for each type of problem that best describes the stiffness of that kind of system (without the effects of the boundary conditions). Then the whole system is divided into a main system and some small subsystems which can be solved separately. Then the effects of those subsystems are introduced to the main system as traction forces. In this way there can be a complete control over the condition number of the main and the subsystems and different mesh sizes can be used in each newly defined problem. This can work properly because of the fact that the nodal displacement values will be accurate enough if the traction forces are evaluated to a suitable accuracy and the used elements can model the pure stiffness of the system.

After evaluating the nodal displacements of the main problem the inside-element values are computed using the predefined subsystems. For a nonlinear problem, this procedure will be applied to the system with the updated values of the variables in each iteration. This method can also be useful in using XFEM method for different problems whose enriched stiffness matrix has noticeable negative effect on the total stiffness matrix of the system.

The introduced procedure has been implemented in a Java based program (Virtual Engineer) and applied for a variety of problems in which the resulted lower condition numbers can increase the reliability of the answers. By using this method, numerical models will become much more useful and can be applied to a wide range of problems.

REFERENCES

[1] H. Kadowaki and W.K. Liu, "Bridging multi-scale method for localization problems", *Comput. Mthods. Appl. Mech. Engrg.*, 193 (2004) 3267-3302.

[2] G.H. Golub and C.F. Van Loan, *Matrix Computations*, 3rd Edition, The Johns Hopkins University Press, 1996.