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A NEW PARAMETERIZATION SCHEME FOR SHAPE OPTIMIZATION PROBLEMS IN PLANE ELASTOSTATICS

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ABSTRACT

The problem of determining and optimizing the effective linear elastic properties of planar structures is a hard and important problem, with wide applications in materials science and engineering. By a structure we mean here a thin elastic plate in which an overall matrix transfers applied loads to periodically located inclusions through a strongly bonded interface. Such composite material shows macroscale (or effective) mechanical properties superior to that of its constituents. At given phases elastic moduli these properties are very sensitive to the geometry of the microstructure and thus can be optimized by appropriate shaping of the inclusions .

For a fixed inclusion shape the effective properties are obtained by averaging the local constitutive equations over the periodic cell. This operation linearly relates the averages of an applied stress field to the induced average strains through the Hooke's matrix of the effective moduli. Taken separately, they describe the structure energy response on a specific trial loading, that is overall pressure (the bulk modulus K_{eff}), pure shear and twisting (the shear moduli $\mu_{eff}^{(1)}$ and $\mu_{eff}^{(2)}$, respectively.)

The effective moduli can be extracted only from the full-scale solution to the boundary-value problem of elastostatics, whose complexity is mainly defined by the inclusion shape. Except in the K_{eff}-minimizing (equi-stress) shapes [1] such extraction requires less computational efforts than finding, say, the local stresses. We use this numerical advantage to develop a robust and simple scheme for energy evaluation of an inclusion shape from the representative set of curves which can be conformally mapped onto a circle by an analytic function with only a small number of non-zero Laurent terms.

Based on the convenient Kolosov-Muskhelishvili (KM) potentials this scheme has novel time saving and accuracy increasing features :
• First, in contrast to the standard KM practice, the potentials are computed here in sequence rather than in parallel, with no singularities involved. This allows the direct determination of the effective moduli from the half-sized resolving system (RS) of linear algebraic system;

• Second, the above-described mapping scheme is employed to economically encode the inclusion shape with the mapping coefficients instead of commonly used representation through a large number of equally spaced nodal points. It should be stressed that the conformal mapping is applied only to the shape and not to the equilibrium equations which are solved to their physical (untransformed) regions;
• In specific situations, the shape mapping permits to take the integral type RS coefficients analytically rather than numerically

The scheme is validated by reference to the literature results for different shapes with hard-to-compute large local curvatures. It is especially effective as a direct solver within an evolutionary shape optimization process. Taken in this case as design variables, the mapping coefficients offer substantial advantages over the nodal points representation:

• They are naturally ordered, that is the higher coefficient, the lesser is its global impact on the strain energy;
• They fall into the successfully narrowing intervals as is resulted from the conformal mapping theory. This allows to treat these intervals as linear constraints in the optimization problem.

Such GA-based optimum searching method has been successfully applied for identifying the $\mu_{eff}^{(1,2)}$-minimizing shape of a single inclusion [2]. Here we extend this approach to the following yet unsolved arrangements:
• *A finite number of energy-minimizing holes. Collective effects and forming a cluster are simulated numerically in dependence on the hole spacing;*
• *Doubly periodic $\mu_{eff}^{(1)}$-optimization problem for a lattice of elastic inclusions. The results obtained cover a wide range of the problem input parameters;*
In concluding we summarize the basic assumptions by which the proposed scheme works well in solving the computationally hard problem of evaluating the elastic effective moduli.
• The problem is 2D rather than 3D. This permits use of an efficient complex variable technique.
• The elastic phases are homogeneous and linearly isotropic with perfect contact along the interface. This admits a simple problem formulation in terms of the KM analytic potentials linked through the boundary conditions.
• The considered structure is doubly periodic. This allows to easily incorporate given loading conditions by the quasi periodic Weierstrassian elliptic functions.
• The effective moduli are computed objects of an averaging rather than local nature. This makes it possible to reach a good accuracy at moderate computational efforts. This is especially important in evolutionary structural optimization when the proposed algorithm is used for assessing a large amount of randomly generated inclusion-matrix interfaces.

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