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A GENERALIZED COSSERAT POINT ELEMENT (CPE) FOR ISOTROPIC NONLINEAR ELASTIC MATERIALS INCLUDING IRREGULAR 3-D BRICK AND THIN STRUCTURES

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ABSTRACT

Recently, Nadler and Rubin [1] developed a 3-D eight noded brick element based on the theory of a Cosserat point. This Cosserat point element (denoted by CPE) can be used to formulate the numerical solution of dynamic problems for nonlinear hyperelastic materials. The kinematics of the CPE are characterized by eight element director vectors and the kinetics propose eight balance laws of director momentum to determine the response of the element. Moreover, the CPE theory considers the element as a structure and introduces a strain energy function which characterizes the response of the structure. In contrast with the standard finite element approach, the CPE needs no integration over the element region, and the nodal forces are related to derivatives of the strain energy function through algebraic relations in a similar manner to the relationship of the stress to derivatives of the strain energy function in the full three-dimensional theory of hyperelastic materials. Restrictions were developed on the strain energy function which ensure that the CPE reproduces exact solutions for all homogeneous deformations [1]. Consequently, the CPE automatically satisfies a nonlinear form of the patch test. Moreover, a functional form of the strain energy for the CPE was proposed with specific dependence on the strain energy of the three-dimensional material and the reference geometry of the CPE element. In addition, a strain energy function for inhomogeneous deformations was introduced as a quadratic function of inhomogeneous strain measures. Loehnert et al. [2] implemented the CPE formulation into the finite element code FEAP and considered specific example problems which showed that the response of the original CPE was robust and locking free for thin structures. However, it was also shown that the accuracy of the original CPE degraded with increased irregularity of the reference element shape. This work generalizes the improved CPE developed in [3] by considering a strain energy function for inhomogeneous deformations that fully couples bending and torsional modes of deformation. Specifically, the coefficients in this strain energy function depend on the reference geometry of the element.

EXAMPLES

A number of examples have been considered to show that the generalized CPE produces results as accurate as enhanced strain and incompatible modes elements for thin structures and is free of hourglass instabilities typically predicted by these enhanced elements in regions experiencing combined high compression with bending [4]. Also, the results of the generalized CPE are shown to be relatively insensitive to irregularity of the reference element shape. The specific examples include: a point load applied to a partially clamped rhombic plate (Fig. 1); a thin circular cylindrical shell subjected to opposing concentrated forces (Fig. 2); and plane strain indentation of a rigid plate into a nearly incompressible elastic block (Fig. 3).

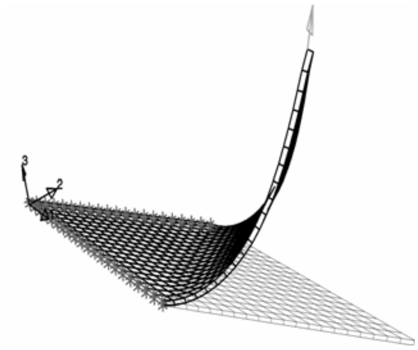

Fig. 1: Point load on a partially clamped rhombic plate (large deformations).

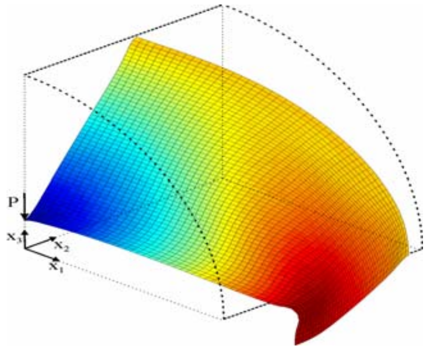

Fig. 2: Deformed shape of an eighth of a thin circular cylindrical shell subjected to a pair of opposing concentrated point loads P.

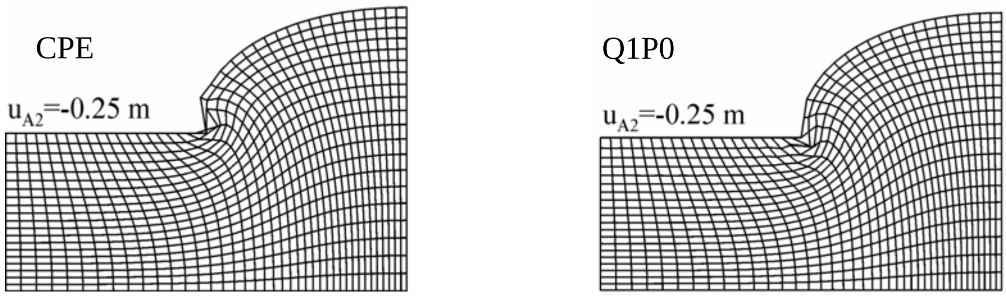

Fig. 3: Plane strain indentation of a rigid plate into a nearly incompressible block showing the deformed shapes for the CPE and Q1P0 elements.

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