# SMOOTHED POINT INTERPOLATION METHODS FOR 2D AND 3D ELASTICITY PROBLEMS WITH CERTIFIED SOLUTIONS

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## ABSTRACT

A class of smoothed point interpolation methods (smoothed PIMs) are introduced in this paper, which are derived from the smoothed Galerkin weak-form for variational formulation based on the gradient smoothing techniques [1]. In the scheme of smoothed PIMs, the strain smoothing operation [2] can be applied on different types of smoothing domains which are constructed centring at field nodes, edges of the background cells or the background cells, and correspondingly different types of smoothed PIMs may be formulated, which are respectively Node-based smoothed point interpolation method (NS-PIM), that was originally termed as linearly conforming point interpolation method (LC-PIM) [3-5], Edge-based Smoothed point interpolation method (ES-PIM) and Cell-based point interpolation method (CS-PIM).

In all these three methods, i.e., NS-PIM, ES-PIM and CS-PIM, the point interpolation method (PIM) is employed to construct shape functions with a small set of nodes located in a local support domain that can overlap each other [6]. For the PIM, it is flexible to construct shape functions using different polynomial terms with corresponding number of supporting nodes. In the present work, the most simple 3-node triangles and 4-node tetrahedron background cells are used to represent 2D and 3D problem domains and a simple scheme is used to find out the support nodes. The constructed PIM shape functions possess the Delta function property, which allows straightforward imposition of point essential boundary conditions. In the smoothed PIMs, the smoothed strains are used over smoothing domains instead of the compatible strains which are obtained from the strain-displacement relation. Based on the background cells, smoothing domains for NS-PIM, ES-PIM and CS-PIM are formed centring at field nodes, edges of the cells and the cells themselves respectively. The discretized system equations of smoothed PIMs are generated based on the smoothed bilinear form for variational formulation, which can be derived from the Hellinger-Reissner's two–field variational principle [1]. The smoothed PIMs' solutions can satisfy

the equilibrium equations locally (free of body force) at any point within the smoothing domains except on the interfaces of the smoothing domains. The displacement fields of the smoothed PIMs are compatible, but the strain fields are not compatible in terms of satisfying the strain-displacement relation. Therefore, the smoothed PIMs behave like a quasi-equilibrium model that combines the equilibrium model and the fully compatible model [4].

A thorough theoretical study on the NS-PIM has been conducted and it has been found and proved that the NS-PIM posses the following important property: it can obtain an upper bound solution in energy norm for elasticity problem with homogeneous essential boundary conditions. We all know that the displacement based fully compatible finite element method (FEM) provides a lower bound solution in energy norm. Then by using FEM together with NS-PIM, we can obtain the lower and upper bounds of the exact strain energy numerically [7].

In the present work, the ES-PIM and the CS-PIM are formulated in the way similar to that of the NS-PIM, intensive 2D and 3D numerical examples are studied and their properties are investigated and compared with those of NS-PIM, especially for the important upper bound property.

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