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AN OVERVIEW OF DEFECTIVE BOUNDARY VALUE PROBLEMS IN HAEMODYNAMICS

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ABSTRACT

In order to perform numerical simulation for fluid-dynamics problems in realistic geometries, often the complexity of the latter forces to consider a {\sl truncated} computational domain in which the Navier-Stokes equations are solved. Whilst on the physical wall it is clear which boundary condition has to be prescribed, on the artificial sections, namely the boundaries created by the truncation of the domain, often only an incomplete set of data is available. In particular, we focus on those problems in which only the flow rate or the mean pressure are known. This is quite typical for instance in haemodymanics, where the common mesurements are able to prescribe only average quantities.

Several startegies with the aim of solving this defective problem have been developed in the literature. The most classical one, intensively used in the engineering comunity, consists in prescribing an arbitrary velocity profile, fitting at each time the flow rate. In this way the defective problem is equipped with a Dirichlet boundary condition and then it is solvable with a classical fluid solver (practical approach). However, the arbitrarity of this choice introduces a bias in the numerical solution, which could be very different from the real one. For this reason, one possibility is to enlarge the computational domain, allowing the velocity profile to develop properly. Obviously, this increases

the computational costs and, however, it is not always able to perform a significant solution.

For this reason in *Heywood et al.* a different, more mathematical approach has been proposed. It is based on writing a suitable variational formulation of the flow rate problem. The drawback of this approach consists in the introduction of non standard finite element spaces.

A third strategy has been proposed in *Formaggia et al. 2002* and then analysed and developed in *Veneziani and Vergara 2005 and 2007* (augmented formulation). It consists in

treating the flow rate conditions as a constraint to be imposed thanks to the introduction of Lagrange multipliers, one for each condition. A particular attention has been given to algorithms based on the calling of pre-existing fluid solvers, used as a black-box.

Finally, in *Formaggia et al. 2008* an approach based on the control theory has been introduced. In particular, the Navier-Stokes equations are considered as a constraint to be forced to the problem of minimization of a suitable functional, which takes into account in some sense the defective conditions. This approach is the ``dual" of the augmented formulation. The nice features of this approach are, again, the possibility of introducing algorithms based on pre-existing fluid solvers and the more versatility in comparison of the augmented formulation. Indeed, different defective conditions, such as the mean pressure conditions, can be treated as well.

REFERENCES

[1] J. Heywood, R. Rannacher, S. Turek, “Artificial boundary and flux and pressure conditions for the incompressible Navier-Stokes equations”, *Int. Journ. Num. Meth. Fluids*, Vol. **22**, pp. 325—352, (1996).

[2] L. Formaggia. J.F. Gerbeau, F. Nobile, A. Quarteroni, “Numerical treatment of Defective Boundary Conditions for the Navier-Stokes equation”, *SIAM J. Num. Anal.*, Vol. **40**(1), pp. 376--401, (2002).

[3] L. Formaggia, A. Veneziani, C. Vergara, “A new approach to numerical solution of defective boundary problems in incompressible fluid dynamics”, *SIAM, J. Num. Anal*, in press.

[4] A. Veneziani, C. Vergara, “Flow rate defective boundary conditions in haemodinamics simulations”, *Int. Journ. Num. Meth. Fluids*, **47**, pp. 803--816, (2005).

[5] A. Veneziani, C. Vergara, “An approximate method for solving incompressible Navier-Stokes problem with flow rate conditions”, *Comp. Math. Appl. Mech. Eng.,* **196**(9-12) , pp. 1685-1700, (2007).