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MIXED FINITE ELEMENT SOLUTION ON THE-OUT-OF PLANE NATURAL FREQUENCIES OF COMPOSITE CIRCULAR BEAMS

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INTRODUCTION AND METHOD

There are many studies on the theory and analysis of beam-type structures in the literature. Curved beam structures have been widely used in many civil, mechanical and aerospace engineering applications such as spring design, curved girder bridges, tire dynamics, stiffeners in aircraft structures. Research in this area can be traced back to the 19th century [1, 2]. Papers dealing with the natural frequencies of curved beams have been published regularly in recent years and many techniques have been considered whether the material is isotropic or composite [3-6]. In this present study, an analysis for the out-of-plane vibration frequencies for different angles (0°, ±30°, ±45°, ±60°, ±75°) of fibers along the axis of a circular orthotropic beam will be given. The effect of shear deformation is taken into account. The Gâteaux differential method (GDM) is employed to construct the functional and variational method [7] is applied to obtain the mixed finite element formulation as done by Akoz and Kadioglu [8].

The governing equations for out-of-plane scalar vibration can be found from literature [9], in which, there are six variables including a shear force (T_b), a bending moment (M_n), a torsional moment (M_t), two components of rotation (Ω_n, Ω_t) and a component of displacement (u_b). Field equations can be written in operator form as $\mathbf{Q} = \mathbf{L}\mathbf{y} - \mathbf{f} = \mathbf{0}$ and are given explicitly in matrix form of the operators. $\mathbf{Q}$ is a potential if the equality $\langle d\mathbf{Q}(\mathbf{y},\bar{\mathbf{y}}),\mathbf{y}^*\rangle = \langle d\mathbf{Q}(\mathbf{y},\mathbf{y}^*),\bar{\mathbf{y}}\rangle$ is satisfied [7] where $d\mathbf{Q}(\mathbf{y},\bar{\mathbf{y}})$ is the Gâteaux derivative of is $\mathbf{Q}$ and the inner product of two vectors. Therefore the functional corresponding to the field equations is obtained as [9]

$$\mathbf{I}(\mathbf{y}) = \int_0^1 \langle \mathbf{Q}(s\mathbf{y},\mathbf{y}),\mathbf{y}\rangle \,ds \tag{1}$$

where *s* is a scalar quantity. Finally from equation (1), the functional of the composite beam is obtained.

$$\mathbf{I(y)} = \frac{\rho\omega^2 AR}{2}[u_b,u_b] + R[T_b,\Omega_n] + [M_n,\Omega_t] - [M_t,\Omega_n] - [T_b',u_b] - [M_n',\Omega_n] - [M_t',\Omega_t] - \frac{RD_{11}'}{2}[M_t,M_t] - \frac{RD_{22}'}{2}[M_n,M_n]$$
$$-\frac{Rk'A_{33}'}{2}[T_b,T_b] + [\hat{u}_b,T_b]_\varepsilon + \left[\hat{\Omega}_n,M_n\right]_\varepsilon + \left[\hat{\Omega}_t,M_t\right]_\varepsilon + \left[(T_b-\hat{T}_b),u_b\right]_\sigma + \left[(M_n-\hat{M}_n),\Omega_n\right]_\sigma + \left[(M_t-\hat{M}_t),\Omega_t\right]_\sigma - RD_{21}'[M_n,M_t]$$

In dynamic analysis, the problem of the natural frequencies of free vibrating structural system reduces to the solution of the standard eigenvalue problem $[\mathbf{K}] - \omega^2[\mathbf{M}] = \mathbf{0}$ where $[\mathbf{K}],[\mathbf{M}],\omega$ is the system matrix, mass matrix and system natural angular frequency, respectively. To derive the finite element formulations, first the interpolation functions, by which all unknown internal quantities have been expressed, must be chosen and then are inserted into the functional. After extremization of this functional with respect to 12 nodal variables, the twelve obtained element equations are solved.

EXAMPLE AND CONCLUSION

The material is chosen as Carbon-Epoxy (IM6/SC1081) and Kevlar-Epoxy for an orthotropic beam. In Table 1, E_i, G_{ij}, ν_{ij} represent the Young's moduli, shear moduli and Poisson ratio, respectively.

Table 1 The transversely isotropic material properties used in this study.

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	ρ (kg/m^3)
Carbon-Epoxy(IM6/SC1081)	177	10.8	7.6	0.27	1600
Kevlar-Epoxy (Aramid149)	87	5.5	2.2	0.34	1380

The shear correction factor is taken to be 6/5. The cross sectional properties of the curved beam are as follows: width=thickness=0.3 m, radius=10 m. Table 2 shows the first three natural frequencies of fixed-fixed ends boundary conditions for different angles (±45°, ±60°, ±75°) of fibers along the circular beam axis are obtained by FORTRAN program and ANSYS software with using element BEAM4.

Table 2 The first three natural frequencies of (Hz) composite beam with fixed-fixed ends

	Carbon-Epoxy (IM6/SC1081)						Kevlar 149-Epoxy					
	Present			ANSYS			Present			ANSYS		
θ	f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3
45°	0.881	2.508	5.254	0.839	2.427	5.008	0.580	1.620	3.374	0.538	1.559	3.261
60°	0,735	2,157	4,515	0,727	2.101	4.392	0.500	1.450	3.030	0.498	1.436	3.008
75°	0.654	1.958	4.112	0.676	1.953	4.079	0.497	1.430	3.003	0.493	1.424	2.988

In conclusion, the formulation presented is general in nature and the method hence may be utilized for general geometry. It gives ease to generate element matrices because of the properties of the new functional. Using the proposed method all kinds of boundary conditions can be analyzed. The element may be used for relatively deep beams. At the same time this formulation can be used for wide and relatively thin structures due to the fact that this formulation is free from locking effects. Natural angular frequency results are compared by solving various simple problems which solutions are available in literature and an agreement is achieved.

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