8th World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

UNIFORM-STRESS CATENARY SIMULATED BY UNSTABLE TRUSS STRUCTURE

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Key Words: *Unstable Truss, Unstable Mode, Equilibrium, Catenary, Optimization.*

ABSTRACT

The catenary is known as the curve of the inextensible string of the specific length with two fixed ends which is subject to the gravity force. Many scholars in the 16th century such as Bernoulli brothers, Leibnitz and Euler studied the catenary from the view points of mathematics and applied mechanics. The original problem of the catenary can be solved by using the cosine-hyperbolic function with three parameters which are to be determined by the given length and the coordinates of the two fixed points of the string. Behroozi et al.[1] extended the analytical solution of the catenary curve to the case where the inward surface tension force as well as the gravity force acts on the string.

There are the two main theoretical approaches to the catenary. One is based on the derivation of the governing differential equation of the catenary directly from the equilibrium condition of the small segment of the string. The other is based on the variational principle which expresses that the catenary has the smallest potential energy among all strings with the common boundaries and length conditions. The above two methods, however, are easily shown to be mathematically equivalent.

The catenaries treated thus far in the literature have the uniform sectional area and therefore they give the widely distributed tensile force and stress along the curves. The object of this paper is to numerically and analytically present the solution of the catenary which possesses the uniform tensile stress along the curve with the varied sectional area. The numerical analysis is done by seeking the equilibrium state of the unstable truss structure under the prescribed loading condition. All members of such a structure are assumed to be perfectly rigid but several number of unstable displacement modes without causing the elongation of the members are found out in the structure. Tanaka et al.[2] studied the stability conditions and stable states of the unstable truss structure considering the concept of the general inverse matrix which seems to make their theory somewhat difficult. Furthermore they did not make any comment to calculate the member forces. If the member forces are obtained, it is not difficult to make the stress distribution uniform by varying the sectional area of each member with the total volume unchanged.

The paper is divided into 5 parts. Firstly, the way to seek the equilibrium state of the unstable truss structure is explained using the elementary transformation of the matrix B which relates the nodal displacements with the member elongations. At the same time, the member forces are proved to be readily calculated using the above transformation with the principle of virtual work. Secondly, a few examples

of the equilibrium state of the unstable truss structures are shown with the member forces confirming the accuracy of the present numerical method. Figure 1(a) shows the initial unstable state(dotted lines) and the final equilibrium state (solid lines) of an unstable truss structure under the gravity force. The two end points are allowed to slide. Figure 1(b) illustrates the member forces in the final state where the dotted lines mean the compressive forces. Thirdly a numerical procedure to make the tensile stress distribution uniform is explained in the case of catenary by changing successively the sectional area along the curve. This is one of the optimization problems of the material distribution. Fourthly the theoretical curve and its distribution of sectional area of catenary with the uniform tensile stress are derived by solving the differential equation. Finally the results are examined as shown in Fig.2. The thick curve in Fig.2(a) illustrates the numerical result of the uniform-stress catenary while the thin curve corresponds to the usual catenary with the uniform cross section. The truss in this example is composed of 30 jointed linear elements. The initial configuration of the unstable truss can be arbitrary. The stress distributions are compared in Fig.2(b) to see the convergence of optimizing process. The theoretical catenary of the uniform stress is depicted in Fig.2(c) with which the numerical one in Fig2.(a) almost coincides.
In the present paper, the emphasis will be put on that the unstable truss structure has a lot of interesting characteristics and is widely applicable to several optimization problems in engineering.

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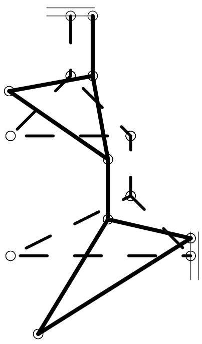

Fig.1(a) Displacements

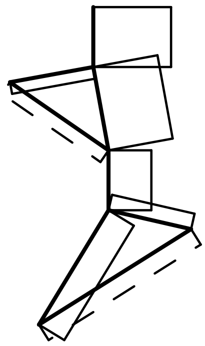

Fig.1(b) Member forces

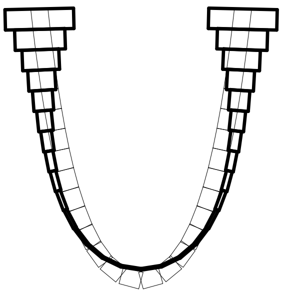

Fig.2(a)Usual and optimized catenaries(numerical)

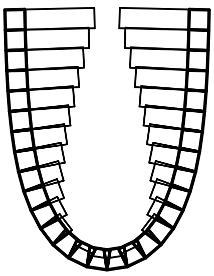

Fig.2(b)Usual and optimized stress distributions(numerical)

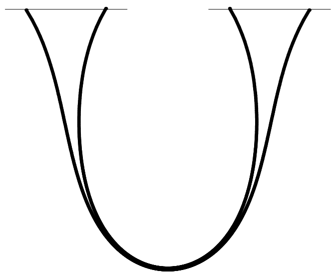

Fig.2(c)Optimum catenary(theoretical)