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INTERPOLATION ALTERNATIVES FOR THE CELL FACE VELOCITY AND THEIR EFFECT ON THE SOLUTION

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ABSTRACT

The management of the pressure-velocity coupling in a colocated grid has always been a source of problems due to the 2-Δ pressure difference in the momentum equations and the 2-Δ velocity difference in the mass conservation equation. If no special care is taken there is no way of obtaining a converged solution as the pressure at a given node is uncoupled from the velocity at the same node. Rhie and Chow [1] adopted an approach named PWIM (Pressure Weighted Interpolation Method) which has been followed by subsequent users of the collocated arrangement whereby the cell face velocity is interpolated from nodal velocities plus an extra term related to the pressure field. Although the nodal pressure is still absent from the momentum equations[1] the pressure is driven by 1-Δ mass imbalance and the whole procedure is stable. In the first applications of this procedure it was observed that when solving the underrelaxed Navier-Stokes equations the solution was unacceptably dependent on the underrelaxation factor. Majumdar [2] and Miller and Schmidt [3] showed that a proper treatment involved the storage of the cell face velocity and the inclusion of its value from the previous iteration in the expression that calculates the current face velocity. It can easily be shown that by doing so the final solution is free from relaxation dependence. For instance, the final expression for the east face is

$$u_e^* = \overline{u_i^*}^e - \overline{\left(\frac{\Delta V_i}{\widehat{A}^u_{P|i}}\left(-\left.\frac{\partial p}{\partial x}\right|_i\right)\right)}^e + \frac{\Delta V_e}{\widehat{A}^u_{P|e}}\left(-\left.\frac{\partial p}{\partial x}\right|_e\right) + (1-\alpha_u)\left[u_e^o - \overline{u_i^o}^e\right] \tag{1}$$

where α_u is the underrelaxation factor, $\widehat{A}^u_{P|i} = A^u_{P|i}/\alpha_u$; $i = E, P$, and the overline represents a mean over the nodal values that share the interface made explicit at the overline, that is

$$\overline{\phi_{.|i}}^e = f_x\phi_{.|P} + (1-f_x)\phi_{.|E} \qquad f_x \text{ is a geometric factor} \tag{2}$$

$A^u_{P|e}$ is the diagonal coefficient of the fictitious east-face velocity equation. When neccesary two subindices are employed: the first one represents the category/localization of the variable and the other specifies the grid location is referred to. The coefficients correspond to the discretization of the original differential equation by a finite volume procedure. Note that in our discretization the terms integrated over the control volume are not divided by the volume itself. This fact will indirectly provide another way of averaging the e-coefficients as explained below.

[1]There exists a slight contribution that produces a loose coupling

The east-face velocity equation is fictitious and as such its coefficients are calculated by averaging over the adjacent nodal values. In particular, $A^u_{P|e}$ can be estimated with the arithmetic or harmonic mean of $A^u_{P|P}$ and $A^u_{P|E}$. Alternatively we could proceed in the same manner with the coefficient per unit volume in the following way

$$A^u_{P|e} = \frac{A^u_{P|e}}{\Delta V_e}\Delta V_e = \overline{\left(\frac{A^u_{P|i}}{\Delta V_i}\right)}^e \Delta V_e \quad \text{with} \quad \Delta V_e = \frac{1}{2}(\Delta V_P + \Delta V_E) \tag{3}$$

One of the issues dealt with in the paper is the effect of employing an arithmetic or harmonic mean for evaluating $A^u_{P|e}$ or $A^u_{P|e}/\Delta V_e$. It will be shown that in certain cases to be described the adopted interpolation is not innocuous to the solution.

The usual PWIM interpolation is just one particular case of a general averaging procedure given by

$$\frac{\widehat{A}^u_{P|e}}{\phi_e}u^*_e = \overline{\frac{\widehat{A}^u_{P|i}}{\phi_i}u^*_i}^e - \overline{\left(\frac{\Delta V_i}{\phi_i}\left(-\left.\frac{\partial p}{\partial x}\right|_i\right)\right)}^e + \left(\frac{\Delta V_e}{\phi_e}\left(-\left.\frac{\partial p}{\partial x}\right|_e\right)\right) + (1-\alpha_u)\left(\frac{\widehat{A}^u_{P|e}}{\phi_e}u^o_e - \overline{\frac{\widehat{A}^u_{P|i}}{\phi_i}u^o_i}^e\right) \tag{4}$$

$\phi_{i,e}$ being an appropriate discrete function. In particular PWIM can be recovered if $\phi_{i,e} = \widehat{A}^u_{P|i,e}$. In the paper it will be shown that depending on the actual ϕ function the solution can present oscillations near the boundaries. In some cases to be decribed there is a need for a special interpolation in thta region, otherwise the rapid variation of the diagonal coefficients produces oscillations that cannot be damped as the grid is refined. The figure presents a comparison with two different ϕ functions, the first one corresponding to the classical Rhie-Chow proposal in a case for which the difference is notable near the right end of the domain. This case corresponds to the one dimensional incompressible Navier-Stokes equations with certain mass and momentum sources.

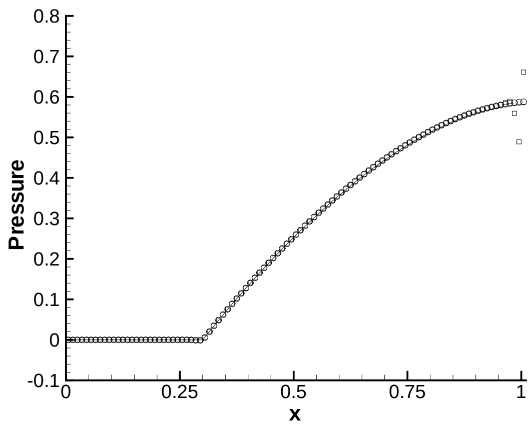

Figure 1: Pressure distribution. ○ Rhie-Chow solution; □ Solution for $\phi = 1$

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