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THE SIGNIFICANCE OF NILPOTENT SENSORS FOR MONITORING IN MULTIFIELD PROBLEMS

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ABSTRACT

In the present paper monitoring in multifield problems is studied within the framework of material bodies with sources of self-stress acting in a linear elastic background body. Multifield coupling is constituted by means of the physical origin of the sources of self-stress; e.g. piezoelectric strains, thermal expansion strains, plastic misfit strains, geometrically non-linear parts of strain or, more general, any type of eigenstrains.

In a purely linear elastic body, nilpotent sensors are characterized as distributed strain-type sensors, which measure a trivial sensor, independent from the actual motion of the body. In three-dimensional problems the distribution of such sensors, their spatial weight, which is represented by a symmetric second order tensor field, can be easily shown to coincide with any possible self-equilibrating stress distribution in the body with no load stresses applied. The latter stress distributions exist in redundant problems only. For a recent contribution to the design of strain-type sensors for monitoring of three-dimensional linear elastic bodies, including a discussion of nilpotent sensors, see Krommer and Irschik [1]. In multifield problems, on the other hand side, nilpotent sensors become sensitive to the presence of any type of eigenstrain; hence, their significance is clearly due to their capability to measure fields other than the linear elastic one. E.g. nilpotent sensors in a piezothermoelastic body can be used to monitor the electric field and/or the temperature in the body.

In particular, we will study the use of piezoelectric sensors for realizing nilpotent sensors. Within the piezoelectric material the electric displacement is coupled to the electric field (electric polarization), the mechanical strain (direct piezoelectric effect) and to the temperature (pyroelectric effect); therefore, a piezoelectric sensor is sensitive to these coupled fields. In the design of piezoelectric sensors, the first effect is typically eliminated by a proper electric circuit (short-circuit, open-circuit or self-sensing), the second effect is used for sensing the deformation of the body and the third is, in most cases, unwanted. We first introduce a general relation for the output of a piezoelectric sensor in the three-dimensional case. Secondly, we derive a representation for the spatial weight of the sensor, such that that part of the general sensor relation, which is related to the linear strain, becomes trivial; hence, represents a nilpotent sensor. The

remaining non-trivial part of the general sensor relation depends on the electric field as well as on the temperature variation. Such nilpotent sensors can then be used to either monitor the latter two fields, or to eliminate their influence on the measured signal of a strain-type sensor.

As structural examples we consider thin-walled structures, such as beams and plates, for which the sensors are put into practice by attaching thin piezoelectric layers and/or patches to the thin substrate structure. The structures are modelled accounting for the multifield electro-thermo-mechanical coupling in the sensing elements within the framework of a classical lamination theory. Sensor relations are derived, nilpotent solutions calculated and their application is discussed. In particular, we consider three important applications.

(1) The use of nilpotent sensors to monitor temperatures. For that sake, nilpotent sensors are put into practice in combination with either open-circuit or short-circuit conditions, such the influence of the electric field on the sensor signal is eliminated.

(2) The elimination of the temperature dependency of the measured signal of a piezoelectric strain-type sensor. For that sake nilpotent sensors are properly superposed upon the strain-type sensors in order to find that sensor distribution that does not measure the temperature. Again, the influence of the electric field on the sensor signal is eliminated by either applying open-circuit or short-circuit conditions.

(3) The use of nilpotent sensors to put self-sensing piezoelectric actuators into practice. In this case the piezoelectric sensor is used as an actuator as well, but the influence of the applied actuator voltage upon the measured signal is eliminated by properly superposing nilpotent sensor distributions.

As special examples redundant beam structures are studied and the validity of the proposed application of nilpotent sensors is checked by analytical and numerical calculations.

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REFERENCES

[1] M. Krommer and H. Irschik, "Sensor and Actuator Design for Displacement Control of Continuous Systems", *Smart Structures and Systems*, Vol. **3**(2), pp. 147–172, (2007).