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A NEW OPTIMIZATION STRATEGY FOR FLOWS IN THE PRESENCE OF SHOCKS: APPLICATION TO THE OPTIMAL DESIGN OF A DUCT.

*** A. Baeza[1], C. Castro[2], F. Palacios[1] and E. Zuazua[3]**

[1] IMDEA Mathematics, Módulo C-IX, Facultad de Ciencias, Universidad Autónoma de Madrid, Madrid, Spain.
antonio.baeza@imdea.org (A. Baeza) and francisco.palacios@imdea.org (F. Palacios)

[2] Dep. de Matemática e Informática, ETSI Caminos, Canales y Puertos, Univ. Politécnica de Madrid, 28040 Madrid, Spain and IMDEA Mathematics, Módulo C-IX, Facultad de Ciencias, Universidad Autónoma de Madrid, Madrid, Spain.
carlos.castro@upm.es

[3] Dep Matemáticas, Facultad de Ciencias, Universidad Autónoma de Madrid, Madrid, Spain and IMDEA Mathematics, Módulo C-IX, Facultad de Ciencias, Universidad Autónoma de Madrid, Madrid, Spain.
enrique.zuazua@uam.es

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ABSTRACT

In this work we adapt a new optimization strategy, introduced in [2] in the context of the one-dimensional inviscid Burgers equation, for one-dimensional systems of conservation laws. In particular we analyze this optimization strategy for the classical problem of optimal shape design of a duct of variable cross-sectional area, by considering steady and unsteady transonic flows with shocks. The dynamics of the fluid is described by the quasi one-dimensional Euler equations.

The problem consists on finding the shape of a duct that optimizes a given cost function, that depends on the properties of the flow that is crossing the duct. Typical cost functions involve integrals of functions of the pressure along the duct, since they represent simplified models for quantities of interest in aeronautical shape design, as thrust, lift or drag. Duct optimization for flows with shocks has been extensively tackled in the scientific aeronautical literature, see e. g. [3, 4].

These problems can be formulated as the minimization of a suitable cost functional, that depends on the pressure distribution of the fluid along the duct, with respect to some admissible design variables. In the one-dimensional model, the duct is assumed to be symmetric with respect to an axis OX and its design is described by the cross sectional area $h(x)$ on a bounded domain $x \in (-1, 1)$. Thus, for steady flows the problem is stated as:

$$\min_{h \in \mathcal{H}} \int_{-1}^{1} g(p)\, dx,$$

where $\mathcal{H}$ is the set of admissible duct shapes; g is some smooth function and p is the pressure of the fluid that we obtain by solving the quasi one-dimensional Euler equations,

$$\begin{cases} \frac{d(hF)}{dx} = \frac{dh}{dx} Q, \qquad x \in (-1,1), \\ + \text{ boundary conditions at } x = -1 \text{ and } x = 1. \end{cases} \tag{1}$$

Here $F = F(u) = (\rho v, \rho v^2 + p, v(\rho E + p))^T$ is the flux vector, which depends on the the vector of conserved variables $u = (\rho, \rho v, \rho E)^T$, where ρ is the density of the fluid, v the velocity and E the energy. The pressure p is given by $p = (\gamma - 1)\rho \left(E - \frac{1}{2}u^2\right)$, where γ is a constant. The vector $Q = (0, p, 0)^T$ is the source term. Boundary conditions in (1) correspond to a transonic flow with a shock wave. Typical cost functions are $g(p) = p$ and $g(p) = \frac{1}{2}(p - p^*)^2$, where p^* is a prescribed pressure distribution to be matched.

We focus on iterative gradient based descent methods, and we use the adjoint approach to obtain the descent directions. The new method is based on a nonstandard gradient calculus, suitable for flows with discontinuities, which takes into account the position of the discontinuities as new design variables. The perturbation of the functional due to a change in the duct design, the linearized Euler system and its associated adjoint are then complemented with the conditions coming from the variations of the new variable, i.e. the shock position. In fact, the idea of taking into account the presence of shock waves in the computation of the gradient of the objective function using the adjoint approach is not new, and has been deduced in several situations before (see [1] or [4] for example). The main contribution here is to take advantage of this calculus in the optimization process. In particular we use this analysis to characterize those variations of the duct shape that do not move the shock position. The set of admissible variations is then decomposed in two subspaces: the one that contains the variations that do not move the position of the shock wave and its complement. In this way, we obtain two classes of descent directions that allow us to implement optimization methods that take advantage of this decomposition, applying the alternating descent algorithm introduced in [2].

The new strategy is very general and can be applied to a large class of optimization problems for conservation laws involving shocks. Moreover, it improves the efficiency of the gradient based methods significantly in some particular situations, especially when the shock position is relevant in the optimization process. In the simpler context of an identification problem for the inviscid Burgers equation, this new strategy avoids oscillations when used in conjunction with a gradient-type optimization method, while improving the efficiency drastically (see [2]).

We will also present some numerical experiments that show that, when the shock position is relevant, the new optimization strategy outperforms typical gradient-based methods that do not consider the shock position as an explicit design variable.

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