8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

MODELLING OF COUPLED ROTATING SYSTEMS WITH GEARS

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Key Words: *Rotating systems, Flexible bodies, Gears, Vibration.*

ABSTRACT

The field of the dynamics of rotating systems has a long tradition and deals with many different types of problems. The typical mechanical systems in rotor dynamics are rotating shafts (or shaft systems) of different shapes joined with special, mostly axi-symmetric, bodies, which can be bladed disks, geared wheels, flywheels etc. These rotating bodies can be coupled together and form complicated mechanical systems like gearboxes and various drive and transmission systems. In the recent years analyzed problems, designed rotating systems and operating conditions are becoming more and more complex. These facts lead also to more complex mathematical models, for which it is sometimes needed to develop new modelling approaches.

The solution of the problems of rotating bodies can be performed either using standard rotor dynamics approaches or using more general multibody approaches. It depends on the considerations related with the problem, mainly on the supposed motion of the investigated system. The modelling approach that is presented in this paper can be used for the modelling of rotating flexible bodies coupled together by gear mesh. It is supposed that the overall motion of bodies consists of large rotating motion with defined constant angular velocity and small vibrations superimposed on this rotation.

The vibration analysis of rotating systems is commonly performed with the assumption of ideal rigid disks. Classic monographs, like [5], describe the modelling of rotors considering disks as rigid bodies with their mass and inertia moments. Rotating flexible shafts are modelled usually on the basis of Bernoulli-Euler or Timoshenko theories. These theories assume that the shaft cross section remains a flat plane and is perpendicular to the centreline during vibration. Effects of rotation are presented in the form of the gyroscopic matrix. A special elastic shaft finite element based on these theories is introduced for example in [7], but rigid disks are still supposed. However, there are cases in which the vibrations of disks become important and the rigid body assumption is too rough for detailed dynamic analysis. The assumption is not correct for example in the case of the high frequency excitation and therefore it is necessary to care about special approaches to the modelling of rotating shafts with flexible disks. Many publications are dedicated to the dynamic analysis of thin rotating disks, e.g. [2, 4, 6]. One of the newest and most comprehensive monographs intended to dynamics of rotating systems is the monograph [3], where the issue of the rotating disks is described in more detail. Started from the

classical membrane theory for thin disks there are shown some possible approaches for modelling general three dimensional rotors and disks considering gyroscopic and centrifugal effects. In all mentioned publications the flexible disk is modelled separately and uncoupled of the shaft subsystem. There is no difficulty in modelling rotors using standard commercial FEM tools, but such codes usually do not take into account all effects caused by the rotation and mainly it is not easy to introduce the special features such as gear mesh couplings.

In this paper the vibration analysis of complex coupled rotating systems is based on the system decomposition into several shaft and disk subsystems. Flexible shafts are modelled using shaft finite elements formulated in the rotating coordinate frame on the basis of the Bernoulli-Euler theory. Disks can be of an arbitrary shape and are modelled as a three dimensional continuum by means of the finite element method. Isoparametric solid finite elements developed in the rotating coordinate frame are used for the discretization of disk subsystems. The presented approach allows the effects of the rotation such as centrifugal and gyroscopic effects to be considered. Flexible couplings between shaft and disk subsystems described by derived stiffness and damping matrices are used.

In order to create the overall model of a whole system also the mathematical model of gear mesh was developed. Standard methodologies are based on the coupling of ideally rigid gears (e.g. [1]) but the gear mesh coupling model presented in this paper can be used between two flexible disks modelled as a three dimensional continuum. Due to the established considerations this approach is not so computationally demanding like complex algorithms used in classical contact mechanics. The modal synthesis method can be efficiently used for the formulation of the whole model and thus the vibrations of the subsystems can be described in the modal coordinates instead of physical ones. The reduced model is then suitable for complex dynamical analyses including optimizations.

The introduced methodology for the modelling of coupled rotating systems is not as general as standard multibody approaches but for most problems of rotor dynamics it is the sufficient simplification. On the other hand this simplification of motion description brings faster calculation times due to the omitting of problematic nonlinear terms in equations of motion. It is possible to use more general gear mesh model characterized by nonlinear coupling. The modelling methodology can be also employed in the dynamical and noise analysis of rotating systems including a compliant stator.

This work was supported by the research project 101/07/P231 of the Czech Science Foundation.

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