## AN APPLICATION OF MESHLESS METHOD IN SHAKE-DOWN ANALYSIS

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## ABSTRACT

The paper deals with the application of a shake-down mechanical model to determine an estimate of residual stresses in prismatic bodies or bodies of revolution subject to cyclic loads of which only a general envelope is known. The discretization of Generalized Finite Difference Method is used, and finally the case is reduced to a large nonlinear optimization problem, solved using a variant of the Feasible Directions Method, having up to 3000 independent decision variables and 650 constraints.

Several accidents [11] or near accidents [5] have occurred during service life of railroad rails and railroad vehicle wheels due to cracking failure. As it was believed, that the residual stress levels in such rails/wheels may have constituted the major driving force behind the development and propagation of cracks, the effort has been undertaken some time ago to offer a viable residual stress determination method taking into account the nature of applied loads, of which only the general envelope is known. Thus the shakedown mechanical model has been proposed [6,7] first for elastic perfectly plastic material and later on generalized to account for kinematic hardening of the material [1]. So far this model, after discretization using the combination of global and local version of the Generalized Finite Difference Method (GFDM), belonging to the rapidly expanding class of meshless methods, has been successfully applied to determine residual stresses in railroad rails and vehicle wheels due to the contact (simulated service) loads only [9,10]. Current research deals with an approach to determine the final residual stress state after the manufacturing (quenching and roller straightening for railroad rail and quenching only for the vehicle wheel) and simulated service.

The initial residual stress distributions (induced during heat treatment and subsequent roller straightening for rail or heat treatment only for wheel) have been determined prior to the current analysis by the staff of US DOT VNTSC [2] as well as the changes in material properties of wheel steel due to high speed braking. The contact loads wandering across the contact surface have been simulated by ten discrete contact locations. Necessary elastic solutions have been calculated using the Finite Difference Method in the rail cross-section and Fourier series expansion along the longitudinal axis (rail) and circumferential direction (wheel) [3,4].

The residual stress distributions obtained so far indicate qualitative agreement with the

experimental results, in terms of both location and size of tensile and compressive stress zones. In terms of numerical efficiency, the method shows clear advantage over standard incremental elastic-plastic analysis in case of relatively small plastic zone, resulting in small to intermediate optimization problem size (up to several hundred decision variables and about 100 active nonlinear constraints). The method yields results for much bigger optimization problems (up to about 3000 decision variables and 650 nonlinear constraints, but at a very substantial time penalty, making its use questionable in such cases.

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