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CONTACT PROBLEMS WITH ANISOTROPIC FRICTION

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ABSTRACT

The contribution deals with solving contact problems with *anisotropic Coulomb friction* for 3D elastic bodies. First we introduce an auxiliary problem with *anisotropic Tresca friction* defining a mapping Φ, which associates with a given slip bound a corresponding normal contact stress in the equilibrium state. A solution to the contact problem with Coulomb friction is a fixed point of Φ and the method of successive approximations can be used for its computation. In a discrete case, it converges for a sufficiently small coefficient of friction [4].

As the Tresca problem is described by a variational inequality of the 2^{nd} kind, its finite element discretization leads to a constrained minimization of a non-smooth function so that a regularization is needed. To this end we use the duality theory that enables us to rewrite the problem in terms of normal contact stresses as a dual minimization of a (smooth) strictly convex quadratic function with separable convex constraints. The solution may be computed by an active set algorithm based on a suitable optimality criterion, e.g. on the KKT-condition [6,8] or on the projected gradient [7]. While the function representing the KKT condition is discontinuous, the projected gradient is continuous that leads to more robust computations. Moreover the algorithm of [7] has a convergence rate in terms of the spectral condition number of the Hessian matrix and it enables easily to incorporate the anisotropic friction into the computational process.

If the FETI domain decomposition method [3] is used, then the corresponding stiffness matrix is positive semidefinite that imposes in addition linear equality constraints in the dual formulation of the Tresca problem. The solution can be computed by an iterative augmented Lagrangian algorithm in which the outer loop updates Lagrange multipliers for the equality constraints, while the inner loop is represented by the algorithm of [7]. Moreover an appropriate penalty update enables us to find the solution at $O(1)$ matrix-vector multiplications. This generalizes scalability results proved originally for frictionless contact problems [1].

Combining the augmented Lagrangian algorithm with the method of successive approximation, we arrive at the iterative scheme for solving problems with Coulomb friction. Computations are considerably more efficient if an inexact implementation is used so that the one successive approximation performs the only step of augmented Lagrangian algorithm. Numerical experiments indicate scalability also of this algorithm.

The dual contact problem with anisotropic Tresca friction discretized by the FETI method reads as:

$$\begin{aligned}
&\text{minimize} \quad \frac{1}{2}\lambda^\top F\lambda - \lambda^\top h, && (1)\\
&\text{subject to} \quad G\lambda = e, && (2)\\
&\qquad \lambda_{\nu,i} \geq 0,\ i = 1,\ldots,m_c, && (3)\\
&\qquad (\lambda_{t_1,i}/p_i)^2 + (\lambda_{t_2,i}/q_i)^2 \leq r_i^2,\ i = 1,\ldots,m_c, && (4)\\
&\qquad \lambda = (\lambda_d^\top, \lambda_g^\top, \lambda_\nu^\top, \lambda_{t_1}^\top, \lambda_{t_2}^\top)^\top,\ \lambda_d^\top \in R^{m_d}, \lambda_g^\top \in R^{m_g}, \lambda_\nu, \lambda_{t_1}, \lambda_{t_2} \in R^{m_c},
\end{aligned}$$

where $F = BK^\dagger B^\top$, $h = BK^\dagger f - c$, $G = R^\top B^\top$, $e = R^\top f$, $r_i \geq 0$ is a slip bound and $p_i, q_i \geq 0$ are anisotropic coefficients at i-th contact node. Here, $K^\dagger$ denotes a generalized inverse to the symmetric positive semidefinite stiffness matrix $K \in \mathbb{R}^{3n_c \times 3n_c}$, $R \in \mathbb{R}^{3n_c \times m_r}$ is a matrix whose columns span the null-space of K, $f \in \mathbb{R}^{3n_c}$ is the load vector, $c = \left(0^\top, 0^\top, d^\top, 0^\top, 0^\top\right)^\top \in \mathbb{R}^{m_d+m_g+3m_c}$ is defined by initial distances $d \in \mathbb{R}^{m_c}$ between bodies and $B = \left(B_d^\top, B_g^\top, N^\top, T_1^\top, T_2^\top\right)^\top \in \mathbb{R}^{(m_d+m_g+3m_c)\times 3n_c}$, where $B_d \in \mathbb{R}^{m_d \times 3n_c}$ enforces the Dirichlet boundary condition [2], $B_g \in \mathbb{R}^{m_g \times 3n_c}$ interconnects parts of the solution [3] and $N, T_1, T_2 \in \mathbb{R}^{m_c \times 3n_c}$ project displacements at contact nodes to normal and tangential directions, respectively [5].

Point out that λ_ν and $\lambda_{t_1}, \lambda_{t_2}$ represent normal and tangential contact stresses, respectively. The anisotropy of the problem is represented by ellipsoidal constraints (3). The implementation of the algorithm of [7] requires to compute orthogonal projections on the ellipses that may be performed applying the Newton method in $\mathbb{R}^2$.

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