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Optimization of lumping schemes for plane square quadratic finite element in elastodynamics

*** R. Kolman[1], J. Plešek[1] and D. Gabriel[1]**

[1]Institute of Thermomechanics
Academy of Sciences of the Czech Republic
Dolejškova 5, 182 00 Praha 8, Czech Republic
e-mail: kolman@it.cas.cz

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ABSTRACT

The numerical solution of the fast transient elastodynamics problem by the finite element method is influenced by the dispersion errors caused by both spatial and time discretizations. For example, numerical attenuation or amplification, polarizations errors, the change of phase and group speeds, numerical diffraction and scattering can be mentioned. Despite of many papers published on the subject, little attention has been paid so far to higher-order elements. Belytschko and Mullen [1] were the first to extend the dispersion analysis to quadratic one-dimensional finite elements. The dispersion study of the three-dimensional second-order Helmholtz equation was carried out by Abboud and Pinsky [2]. In Reference [3] recent results accomplished by the authors are summarized, in particular the extension of dispersion analysis to the eight-node serendipity finite elements. The outcome of the study includes spatial dispersion diagrams, time-space diagrams set up both for explicit (central difference method) and implicit (Newmark) integration methods as well as the Courant number-mesh size indicator H/λ error contour maps.

In this work, we focused on a detailed analysis of the mass matrix lumping schemes for plane square quadratic eight-node elements with variable mass distribution using the results of the aforementioned study [3]. To motivate this work one can notice that the effectiveness of explicit direct time integration methods is conditioned by using diagonal mass matrix, which entails significant computational savings and storage advantages. In the past decades several procedures that produced diagonally lumped mass matrices were developed. For example, the row sum method and diagonal scaling method (Hinton-Rock-Zienkiewicz or HRZ) [4] can be mentioned.

In general, mass matrix must satisfy certain conditions such as symmetry, conservation and positivity which ensure its physical admissibility precluding numerical instability nuisances. Therefore, the diagonal components of the lumped mass matrix must be positive. Furthermore, masses corresponding to the corner nodes and masses corresponding to the midside nodes coincide for the plane eight-node element. The condition for the conservation of the total element mass yields $m = 4(m_1 + m_2)$, where

m_1 and m_2 denote the mass of the midside node and the corner node, respectively. If the mass m_1 is proportional to the total element mass m as $m_1 = xm$, then it holds for the mass $m_2 = (0.25 - x)m$, where x is the mass parameter.

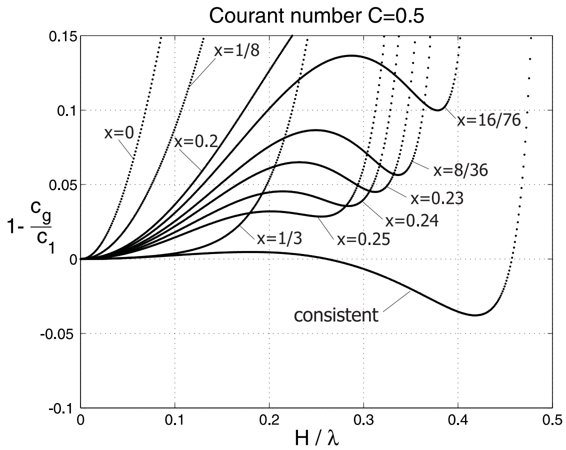

Figure 1: Dispersion diagram of different lumped matrices with variable mass distribution for plane square eight-node element (Co $= 0.5$).

Comparison of dispersion properties of different lumped matrices with variable mass parameter x follows from Fig. 1, where the relative error of the group speeds $1 - c_g/c_1$ versus the normalized wave length H/λ is drawn for the dimensionless Courant number Co $= 0.5$. The value $x = 8/36$ corresponds to the HRZ procedure with 2×2 Gauss quadrature, $x = 16/76$ to the HRZ procedure with 3×3 Gauss quadrature and $x = 1/3$ to the row sum method. It is clear that various nodal mass distribution strongly influences dispersion properties of the lumped matrices. For example, it is shown that the HRZ mass ratio $x = 16/76$ is far from optimum. On the contrary, the most accurate solution is surprisingly obtained for $x = 0.23$ mass ratio when $92\ \%$ of total mass is coalesced into four midside nodes.

Note that this method can be directly applied to the rectangular eight-node element, where the mass coefficients in each axis are chosen proportional to the aspect ratio. Furthermore, the proposed technique can be generalized for the quadrilaterals of arbitrary shape using the piecewise constant shape functions in the derivation of the lumped mass matrix of the unit square element so that it holds $x \geq 0.23$. For the extension to the 9-node Lagrangian element it is necessary to introduce additional mass parameter for the middle-centred node whose determination is given by the fulfilment of the conservation of angular momentum of element.

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