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MATHEMATICAL MODELLING AND OPTIMAL CONTROL OF THE SOLIDIFICATION PROCESS

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ABSTRACT

Mathematical simulation is a key step in the study of various practically important problems. It helps us gain deeper insight into the underlying phenomena and choose control parameters so that the studied process proceeds according to a preset scenario or similar to it.

Problems concerning the optimal control of thermal processes are among those that are difficult to study without resorting to mathematical simulation.

An important class of heat transfer problems is that describing processes in which the substance under study undergoes phase transitions accompanied by heat release or absorption (Stefan problems). A key feature of these problems is that they involve the moving interface between two phases (liquid and solid). The law of motion of the interface is unknown in advance and has to be determined. It is on this interface where heat release or absorption associated with phase transitions occurs. The thermal properties of the substance on the different sides of the moving interface can be different.

We consider an important and interesting problem of this class, namely, the optimal control of the process of melting and solidification in metal casting. For metal crystallizing the special device is used. It consists of upper and lower parts. The upper part consists of a furnace with a mold moving inside. The lower part is a cooling bath consisting of a large tank filled with liquid aluminum whose temperature is somewhat higher than the aluminum melting point. The cooling of liquid metal in the furnace proceeds as follows. On the one hand, the mold is slowly immersed in the low-temperature liquid aluminum, which causes the solidification of the metal. On the other hand, the mold gains heat from the walls of the furnace, which prevents the solidification process from proceeding too fast. So different parts of the mould external bounder are in different heat conditions, depending upon time. The crystallization process is affected by different phenomena such as heat losses due to its radiation, obtaining of energy by a mould due to expanse of radiation from aluminum and furnace, heat exchange between liquid aluminum and the mould. The complication of the task is

that the metal can be present at considered conditions simultaneously in two phases: solid and liquid.

The process of melting and solidification in metal is modeled by a three-dimensional, two-phase, initial–boundary value problem of the Stefan type. A numerical algorithm is presented for solving the initial–boundary value problem [1]. The finite-difference approximation of this problem is based on the Peaceman–Rachford scheme. Primary attention was given to the evolution of the solidification front. The evolution of the solidification front is affected by numerous parameters (for example, by the furnace temperature, the liquid aluminum temperature, the depth to which the object is immersed in the liquid aluminum, the speed at which the mold moves relative to the furnace, etc.). The solidification front as a function of the velocity of the object is of special interest in practice.

The optimal control problem is to choose a regime of metal cooling and solidification in the furnace to be such that the solidification front has a preset or nearly preset shape (namely, a plane orthogonal to the vertical axis of the object) and moves sufficiently slowly (at a speed close to the preset one). The time-dependent speed of movement of the mould along the furnace was selected as the control. The cost functional was chosen as the mean of the standard deviation of the substantial surface of crystallization from the desirable one during the process.

The control function was approximated by a piecewise constant function. As a result of such approximation the cost function is the function of finite number of variables. The minimum value of a cost function was found numerically with use of gradient methods. The gradient of the cost function was found with the help of the finite-difference method and with the help of conjugate problem. The discreet conjugate problem was posed with the help of Fast Automatic Differentiation technique [2].

The problem was primarily studied for an object of the simplest shape – a parallelepiped. This object was used to test and tune the algorithms proposed for solving the problem. The problem was also solved for an actual object of practice interest. The results are described and analyzed in detail.

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