

Fundamental Study on Shape Optimization of a Rigid-Plastic Structure

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ABSTRACT

Rigid plastic analysis is a classical and simple method to evaluate the ultimate load capacities and their corresponding failure mechanisms of structures. Kobayashi has proposed a hybrid type rigid plastic finite element method (Hybrid RPFEM) based on variational inequalities [1]. Since an optimization algorithm of the Hybrid RPFEM is the primal dual interior point method, both static and kinematic constraint conditions are solved simultaneously during the iterative calculations, while decreasing a duality gap gradually from positive value to zero. The advantage of this method is numerical stability and fast calculations. It is also advantageous that additional equality / inequality constraints are easily implemented, because a new formulation is directly derived from just a simple modification of its Lagrangian.

As a variation of this Hybrid RPFEM, Saito et al. have proposed a numerical optimization of strength distribution in a rigid plastic body subjected to a constant load [2]. In their paper, reinforcement of a soil is considered to improve bearing capacity. The objective function of their formulation is a cost which is a linear function of material strengths. To obtain a minimum cost solution, the optimized material strength distribution which can hold a given external load is searched for in the iterative calculations.

On the contrary of a previous study [2], in this paper, subtraction of parts from an initial shape is considered to find the optimal shape of a structure which can carry a given external load. To this end, an auxiliary strength distribution optimization problem is formulated by introducing a log barrier type merit function which plays a key role to select as smaller material strength as possible. In this static formulation, a merit is maximized with satisfying the equilibrium of forces and yielding conditions. A new Lagrangian is proposed based on this maximization problem. With the help of Lagrangian duality theory, both primal (static) and dual (kinematic) optimization problems are derived from the proposed Lagrangian.

In addition to the theoretical proposal, hybrid type formulation based on the primal-dual interior point method is also derived to solve this problem efficiently. According to the hybrid type formulation, a numerical code is developed to check its basic performance and numerical stability. Some numerical results are shown and discussed.

REFERENCES

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