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A discontinuous SPH formulation and application for 2D and 3D problems

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ABSTRACT

When multilayered problems with large deformation are simulated by the SPH method, the physical quantities as density and strain at the material interface are often discontinuous. So it is necessary to introduce the discontinuous SPH (DSPH) method. D.F. Medina[1] realized the discontinuous problem and used the ghost particle technique to describe the discontinuous strain at the composite interface, but they didn't give any recommendation on the dealing. M.B.Liu[2] proposed one-dimensional DSPH formula to describe the discontiuity and found it gave good results. However, they pointed out that it is difficult to determine the discontinuous interface and the key point x_k to perform a real application. That is the objective of this research.

First of all, two-dimensional and three-dimensional DSPH expressions are derived based on the original idea of M.B.Liu[2]. In the field of solid mechanics, the discontinuous interface can be simplified as the materical interface, so the difficulty to find the discontinuous interface is avoided. The main contribution of this research is to introduce a convinent way to perform the DSPH to 3D problem.

The first step of the way is to find the discontinuous interface during every time step. In the compuated domain if the material of an evaluated particle is different from the material of its surrounding particles, the material interface would be located between the two. Then the DSPH expression should be applied to this evaluated particle. The second step is to determine the key point x_k in the discontinuous expression. In one-dimensional DSPH simulation[2], x_k is recommended to be taken as the most adjacent particle to the interface with different material against the evaluated one. However in real simulation it is hard to find the point x_k especially for large deformation, because particles moved very irregularly during the deforamtion. Here, a convinient way to determine x_k is derived based on the Taylor series expansion. In one-dimensional DSPH simulation[2], the computed domain Ω is divided into two different material domains Ω_1 and Ω_2 . In our research the domain Ω_2 is re-divided into many small subdomains $\Omega_{2,1},\cdots,\Omega_{2,n}$ and let only one particle is included in each. Suppose the particle inside each subdomain is named $x_{j1,\cdots}x_{j2,\cdots,}x_{jn}$, respectively, the integral of the function on Ω can be expressed as

$$\int_{\Omega} f(x)W_i(x)dx = \int_{\Omega_1} f(x)W_i(x)dx + \int_{\Omega_{2,1}} f(x)W_i(x)dx + \int_{\Omega_{2,2}} f(x)W_i(x)dx + \cdots + \int_{\Omega_{2,n}} f(x)W_i(x)dx\,.$$

Implemented Taylor expansion in each small subdomain, x_k can only be taken as $x_{j,t}(t=1,2,\cdots n)$ in each subdomain. That is, for every particle with different material from that of the evaluated particle, its corresponding x_k is taken as itself. This way to determine x_k is simple and can be easily applied in large deformation.The feasibility of the new way is validated numerically and Table 1 shows the comparison of different ways of choosing the key point x_k .

Table 1 Comparison for different x_k determinate ways

x_k	function	8	9	10
$x_k = x_{k1}$	f	1.1783%	-0.688%	-6.607%
	$\partial f/\partial x$	0%	-2.222%	-4%
$x_k = x_{k2}$	f	1.1783%	0.2429%	-0.197%
	$\partial f/\partial x$	0%	-2.778%	-5%
$x_k = x_j$	f	1.1783%	-0.688%	-5.852%
	$\partial f/\partial x$	0%	-2.222%	-3.5%

2D and 3D discontinuous functions are tested respectively by the initial SPH method (ISPH), CSPM[3] and DSPH method. The results are shown in Fig1 (a) and (b), which are the comparisons of function and its first derivative respectively. It is clear that the DSPH method is most appropriate for the description of discontinuous quantities.

Further, the sensitivity of DSPH method is discussed and analyzed. It is found that when the function gradually changes from the discontinuity to the continuity, the DSPH behaves worse than ISPH. It is thought there maybe some conditions to judge the application of the DSPH method.

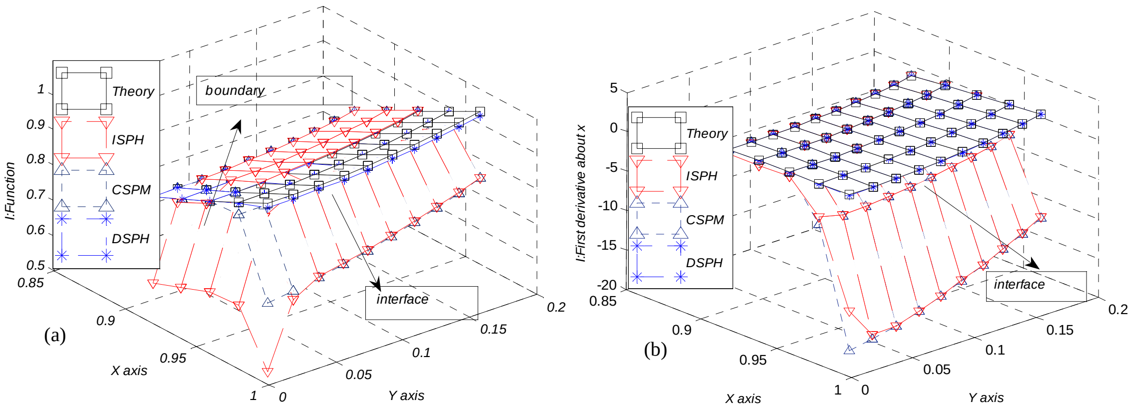

Fig.1 2D Comparison for discontinuous quantities simulations by different methods

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