8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

ABSTRACT TITLE

Asymptotic derivation of section averaged models for river hydraulics: a second order approximation with generalized friction term

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Key Words: *Computational hydraulics, shallow water equations, Saint Venant equations.*

ABSTRACT

Section-averaged shallow water models usually applied in river hydraulics, are derived asymptotically up to second order, starting from the three-dimensional Reynolds-averaged Navier-Stokes equations for incompressible free surface flows. As a result, a generalized friction term is obtained, which is analogous to those introduced in empirical closures widely applied in computational hydraulics (see e.g. [3]).

When the viscosity is neglected and a rectangular channel section is assumed, the derivation of the one-dimensional Shallow Water system is classical [1]. However, this derivation is unsatisfactory, since viscosity effects are added *a posteriori* and the three-dimensional geometry is not arbitrary. In order to simulate realistic river flows, three-dimensional geometries and turbulence phenomena must be taken into account.

Following Gerbeau and Perthame in [2], we propose a rigourous derivation of the section-averaged system including the effects of eddy viscosity, friction and arbitrary three-dimensional geometry. This derivation is also aimed at providing an adequate framework for the rigorous derivation of coupling between three- and one-dimensional free surface models. The resulting section-averaged equations approximate the original three-dimensional system up to the second order in the ratio between vertical and longitudinal scales. This novel derivation yields a generalized friction term that is similar to that of the classical section-averaged shallow water model [3], but includes a correction depending on the turbulent vertical viscosity model. This conclusion is in good agreement with the one achieved in [2] when the vertical viscosity is taken constant and the flow is homogeneous in the transversal direction. Furthermore, the friction term derived here could replace the empirical closures usually applied in hydraulic models with a more general formulation based on rigorous asymptotics.

Steady state analytic solutions for open channel flow have been computed for the derived model, obtaining solutions that are much closer to those of the three-dimensional model than the solutions computed

by the classical one-dimensional shallow-water models. The friction correction term can be easily included in section averaged models such as the one proposed by Deponti *ea* in [4]. Its use is also expected to ease the coupling of three- and one-dimensional free surface models in the framework of an integrated hydrological basin model.

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