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BAYESIAN MODELLING ERROR APPROACH FOR UNCERTAINTIES IN THE MODEL PARAMETERS—EXPERIMENTAL RESULTS IN ELECTRICAL IMPEDANCE TOMOGRAPHY

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ABSTRACT

In inverse problems, the unknown quantity is estimated based on the indirect measurements. Inverse problems are known to be extremely sensitive to measurement and modelling errors. There are several approaches for the measurement uncertainties. However, the modelling errors are often ignored in the estimation process despite the fact that in some cases the modelling error level exceeds the measurement error level.

In reality, the modelling errors are very hard to avoid, since practical applications contain features that are difficult to model. For example, in process monitoring applications image reconstruction must be computed in real time, and the time constraint dictates that reduced models have to be used. The computational models are often based on some numerical scheme such as finite element method (FEM). Thus model reduction might be done by using reduced discretization, for example. Furthermore, the mathematical model, in which the numerical scheme is based on, may contain poorly known parameters or the value of the parameter may be time-dependent. In this paper, uncertain parameters in the forward model are studied in the case of electrical impedance tomography (EIT).

In EIT, alternating electrical currents are injected into the object through boundary electrodes, and corresponding voltages on all electrodes are measured. The conductivity of the object is estimated based on the measured voltages and known currents. The estimation of conductivity is an ill-posed non-linear inverse problem. The contact impedances between the electrodes and the object affect significantly to the measured voltages, therefore contact impedances are taken into account in the forward model. Usually it is thought that the contact impedances are time invariant and equal on the all electrodes. In reality, these assumptions are not true due to contamination, for example.

It is well known fact that the measurements done from current carrying electrodes are most sensitive to contact impedance. Thus these measurements are often omitted, which can be done if relatively small

number of electrodes is used for the current injection. In practical applications with limited computational time optimal current injections are preferred, since the number of current injections required for good reconstruction is smaller. In optimal current injections all or almost all electrodes are used for current injection. The estimation of the unknown contact impedances is also a feasible approach, although not possible if online reconstructions are required [3]. Thus the uncertain contact impedances are a major problem in image reconstruction.In this paper, trigonometric current patterns are used.

Recently, a Bayesian modelling error approach has been proposed [1,2]. In the Bayesian framework, all parameters are modelled as random variables. In this approach, the uncertainty in the measurements and modelling can be separated, and the degree of uncertainties can be estimated. More precisely, the measurement and modelling error are treated as random variables and the degree of uncertainty of both variables is expressed in their distributions. The estimation of the modelling error distribution is done before the measurements, the computational burden using conventional and modelling error approach is therefore approximatively equal. The approach have been applied earlier to number of problems [2], however, in this paper the application to uncertain contact impedances is done for the first time.

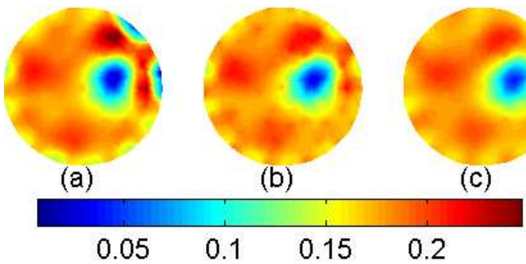

Figure 1: Estimated conductivity distributions within the tank. (a) Contact impedances estimated as a single parameter. (b) Contact impedances estimated separately for all the electrodes. (c) Contact impedances estimated as a single parameter and the modelling error approach is used.

In the experimental test, a cylindrical plastic rod is placed into a water tank, and sixteen electrodes are attached on the surface of the tank. Insulating vaseline is dabbed on the surface of two electrodes. The computed reconstructions (based on the same measurement) are shown in the Figure 1. As can be seen from the Figure 1 (a) the reconstruction error is strong near the contaminated electrodes when only a single parameter for the contact impedances is estimated. It can be seen that the estimation of all contact impedances for all electrodes (b) and the modelling error approach (c) gives comparable reconstructions. It can be concluded that reconstruction errors due to uncertain contact impedances can be reduced by a large factor using the modelling error approach. Moreover, the reconstruction quality is as good as in the more computationally demanding approach in which the contact impedances are estimated on all electrodes. It should be noted that the modelling error approach is general approach and can be applied to any corresponding inverse problem with uncertain model parameters as well.

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