

## DISCONTINUOUS GALERKIN SOLUTION OF THE EULER EQUATIONS USING AN $h$ -MULTIGRID METHOD

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### ABSTRACT

This paper presents an  $h$ -multigrid method for the solution of the high-order discontinuous Galerkin (DG) discretization of the Euler equations. The method applies to general unstructured grids whereby each coarse grid is constructed by agglomerating elements of the next finer grid.

In contrast to the growing number of papers devoted to the  $p$ -multigrid technique in DG methods,  $h$ -multigrid has received far less attention in the DG context [1]. One reason for this is that  $p$ -multigrid perfectly fits in the DG framework, whereas  $h$ -multigrid cannot be easily implemented in DG formulations based on usual (nodal or modal) polynomial approximations on reference elements.

In this paper we describe the main building blocks of our  $h$ -multigrid DG method. Firstly we briefly overview our approach to high-order DG approximation and integration on generic polyhedral elements [2] and then we present the grid transfer operators and the semi-implicit Runge-Kutta smoother. The results of numerical experiments will show the effectiveness of this method for high-order DG solutions of steady Euler problems. In particular, we will focus on studying the performance of our  $h$ -multigrid DG method with respect to: i) the degree of polynomial approximation (kept fixed at all grid levels) and ii) the ratio between number of elements on different grid levels.

Numerical results for the flow around a NACA 0012 airfoil will be provided in the full paper. The performance of the  $h$ -multigrid technique will be demonstrated for polynomial approximations up to  $\mathbb{P}_6$ .

### REFERENCES

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