

# AUGMENTED MACRO-HYBRID MIXED FINITE ELEMENT SCHEMES FOR ELASTIC CONTACT PROBLEMS

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## ABSTRACT

On a setting of subdifferential models, variational augmented macro-hybrid mixed finite element schemes are formulated and analyzed, for elastic unilateral contact problems with prescribed friction. Composition duality principles determine primal and dual mixed solvability, adopting coupling surjectivity for dualization. Macro-hybridization corresponds to nonoverlapping decompositions of elastic solid body systems, with displacement continuity and traction equilibrium transmission conditions dualized. Then, in general, traction and displacement multipliers synchronize sub-bodies through nonmatching finite element interfaces. Three-field formulations give the basis for variational augmentation, in a sense of exact penalization, leading to proximation procedures of parallel numerical resolution algorithms.

In the approximation of constrained problems in mechanics, mixed finite element methods have proved their efficiency in handling constraints via dualization and offering accurate numerical simulation for both primal and dual fields. Such discretization schemes, based on mixed variational formulations of the problem, usually may be of a primal or a dual type according to the variational role played by the balance, constitutive and constraint equations of the mechanical system. In the context of contact problems of solid mechanics, primal mixed variational formulations model displacement-balance as a primal component and contact traction constraints as a dual component, while dual mixed variational formulations model stress-constitutivity as a primal component and displacement-balance and contact displacement constraints as a dual component. Importantly, these two mixed formulations are not in duality, they are different variational models that may be complementary for analysis and computation.

Further, macro-hybridization as a mixed variational formulation where systems are conceived in terms of sub-systems synchronized across their interfaces, permits the computational treatment of big systems, with complex geometry, discontinuous parameters and different local

constitutivities [1]. All of these, done on the basis of nonoverlapping domain decompositions with dualized primal transmission constraints, have in addition the natural advantage of being implementable on nonmatching finite element meshes. In the case of contact mechanical problems, primal or dual mixed families of local variational models are produced, synchronized by subdomain internal boundary tractions and displacements as Lagrangian multipliers, respectively.

One more variant of mixed variational models is their own three-field reformulation. This expanded variational mixed formulation that incorporates intermediate primal fields by dualization, allows the derivation of preconditioned augmented problems as exact dual penalizations. Furthermore, three-field versions of macro-hybrid mixed variational formulations lead to proximation procedures of parallel numerical resolution algorithms [2]. Regarding unilateral contact problems, the intermediate primal fields correspond to the contact normal displacement in the primal mixed problem, and to the stress divergence and contact pressure in the dual mixed problem.

Our interest in this paper is to analyze augmented macro-hybrid mixed finite element schemes for elastic unilateral contact problems with prescribed friction. A distinctive aspect of the study is its non classical approach, formulating and analyzing variational problems on a setting of monotone subdifferential variational mixed models. This multivalued treatment of constrained problems permits the application of composition duality principles, based on coupling surjectivity as the central mixed compatibility condition [3]. Then, for the primal mixed contact problem the surjectivity of the coupling interior-boundary displacement operator is demanded, while for the dual mixed contact problem the surjectivity of the coupling interior stress-boundary traction operator and the coupling interior stress-stress divergence operator are instead demanded. Composition duality principles determine primal and dual mixed solvability in relation with corresponding multivalued primal and dual variational problems.

We shall consider specially three-field formulations of both primal and dual mixed elastic contact problems, clarifying their differences and advantages as variational and computational models. Following the strategy of our previous study on linear mixed problems [4], we translate monotone subdifferential continuous and discrete augmented three-field macro-hybrid mixed contact problems into classical mixed variational inequalities on product functional frameworks. Then, error estimates and mixed finite element rates of convergence are derived as corollaries accordingly to standard mixed analysis.

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