

## A SIMPLE PROCEDURE FOR ANALYSIS OF HYPERELASTIC 3D CABLE STRUCTURES

\*Katalin K. Klinka<sup>1</sup>, Vinicius F. Arcaro<sup>2</sup>

<sup>1</sup> BME, College of Civil Engineering  
2 Bertalan Lajos, Z Building  
Budapest, H-1111, Hungary  
<http://www.klinka.hu/katalinklinka>  
katalin@klinka.hu

<sup>2</sup> UNICAMP, College of Civil Engineering  
Avenida Albert Einstein 951  
Campinas, SP 13083-852, Brazil  
<http://www.arcaro.org/>  
vinicius@arcaro.org

**Key Words:** *cable, hyperelastic, minimization, nonlinear, optimization.*

### ABSTRACT

This text presents a mathematical modeling of a linear line element. The deformation gradient tensor is written in terms of nodal displacements. The invariants of the Right Cauchy-Green deformation tensor are written in terms of nodal displacements. The total potential energy is minimized using a quasi-Newton method. In case of an incompressible material, the incompressibility constraint is satisfied exactly avoiding difficulties in the numerical simulation. In case of a compressible material, the total potential energy is minimized with respect to the nodal displacements and element thickness.

The idea of minimizing the total potential energy to find equilibrium was first introduced by reference [1] for cable network analysis. The advantages of this approach are: It is not necessary to derive the stiffness matrix; it is not necessary to solve any system of equations; it allows a simple static analysis instead of a pseudo-dynamic analysis (dynamic relaxation with kinetic damping).

The computer code uses the limited memory BFGS to handle large scale problems as described by reference [3]. It also employs a line search procedure with safeguards as described by reference [2]. The source and executable computer codes are available for download from the authors' websites.

### Deformation gradient tensor

Figure 1 show the geometric entities used in the mathematical expressions.

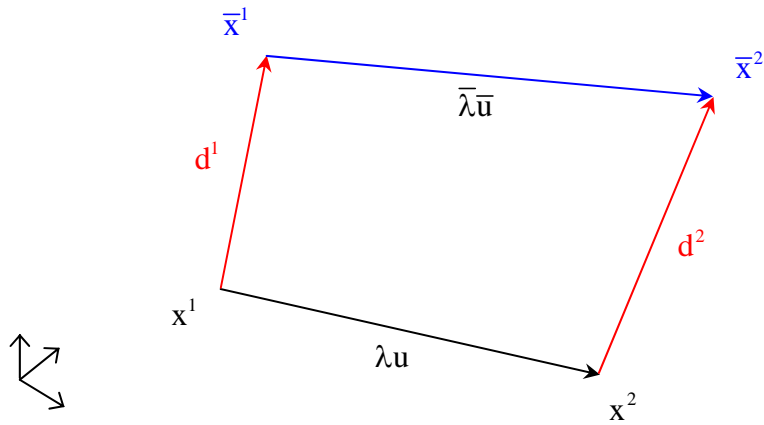


Figure 1

Consider unit vectors  $u$  and  $\bar{u}$  parallel to the element in the undeformed state and deformed state respectively.

Consider the scalars  $\delta$  and  $\bar{\delta}$  equal to the thickness of the element in the undeformed state and deformed state respectively.

Consider unit vectors  $v$  and  $w$  orthogonal to the element in the undeformed state, such that:

$$w = u \times v$$

Consider unit vectors  $\bar{v}$  and  $\bar{w}$  orthogonal to the element in the deformed state, such that:

$$\bar{w} = \bar{u} \times \bar{v}$$

The deformation gradient tensor can be written as:

$$F = I + \frac{(d^2 - d^1)u^T}{\lambda} + \left( \frac{\bar{\delta}}{\delta} \bar{v} - v \right) v^T + \left( \frac{\bar{\delta}}{\delta} \bar{w} - w \right) w^T$$

Notice that,

$$F(\lambda u) = \bar{\lambda} \bar{u}$$

$$F(\delta v) = \bar{\delta} \bar{v}$$

$$F(\delta w) = \bar{\delta} \bar{w}$$

## REFERENCES

- [1] J. P. Coyette and P. Guisset, "Cable network analysis by a nonlinear programming technique", *Engineering Structures*, **10**, pp. 41-46, (1988).
- [2] P. E. Gill and W. Murray, "Newton type methods for unconstrained and linearly constrained optimization", *Mathematical Programming*, **7**, pp. 311-350, (1974).
- [3] J. Nocedal and S. J. Wright, *Numerical Optimization*, Springer-Verlag, 1999.