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ANALYSIS OF CRACKED ORTHOTROPIC MEDIA BY AN ENRICHED FINITE POINT METHOD (EFPM)

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ABSTRACT

The increasing use of composite materials in many engineering applications has motivated researchers to study their damage and fracture behaviours in the past decade[1, 2]. The reason is that fracture mechanisms of composites are different from that of traditional isotropic materials due to their anisotropy and nonhomogenity, which can lead to complicated crack tip singularities and non-plane crack extensions. Among powerful classical numerical methods, meshless methods have largely contributed to the recent advances of fracture mechanics[3]. Among them, the finite point method (FPM) is a truly meshless method which uses a moving least square approximation within a collocation strong form for solving the governing differential equation[4]. In collocation methods, the discrete equations are obtained by enforcing the equilibrium equation on a set of predefiend nodes. Here, a new modified FPM formulation for fracture problems is proposed by augmenting the classical basis of FPM by a set of analytical asymptotic fields. The approximation of the function has the form of:

$$u(x) \cong u^h = \sum_{i=1}^{m} p_i(x)\alpha_i = p^T(x)A^{-1}B\bar{u} \tag{1}$$

One way to enrich the FPM formulation for modeling singular stress fields at a crack tip is to include the leading terms of the near-tip asymptotic expansion for the displacement field in the basis function. The entire near-tip asymptotic displacement terms for an orthotropic material are added to the linear terms of the basis (parameters are defined in[1]):

$$p^T(x) = \left[1, x, y, \sqrt{r}\cos\frac{\theta_1}{2}\sqrt{g_1(\theta)}, \sqrt{r}\cos\frac{\theta_2}{2}\sqrt{g_2(\theta)}, \sqrt{r}\sin\frac{\theta_1}{2}\sqrt{g_1(\theta)}, \sqrt{r}\sin\frac{\theta_2}{2}\sqrt{g_2(\theta)}\right] \tag{2}$$

$$g_k(\theta) = \sqrt{(\cos\theta + \mu_{kx}\sin\theta)^2 + (\mu_{ky}\sin\theta)^2}, \theta_k = tg^{-1}\left(\frac{\mu_{ky}\sin\theta}{\cos\theta + \mu_{kx}\sin\theta}\right), (k = 1,2) \tag{3}$$

In order to evaluate the strain intensity factore, the concept of J integral is adopted. The two-dimensional form of the J integral can be written as:

$$J = \oint_\Gamma (wdy - \mathbf{t}\frac{\partial u}{\partial x}d\Gamma) = \oint_\Gamma (wn_x - n_j\sigma_{ij}\frac{\partial u_i}{\partial x}d\Gamma), \qquad w = \int_0^\varepsilon \sigma_{ij}d\varepsilon_{ij} \tag{4}$$

where w is the strain energy density, Γ is a closed contour, $\mathbf{t}$ is the traction vector on a plane defined by the outward normal $\mathbf{n}$ and $\mathbf{t} = \sigma\mathbf{n}$, $\mathbf{u}$ is the displacement vector, and $d\Gamma$ is the

element of the arc along the path Γ. In a linear elastic analysis, and under pure mode I, the strain intensity factor K_I can be approximately defined as:

$$K_I = \sqrt{EJ_1} \quad (plane \quad stress), E = \sqrt{E_1 E_2} \text{ (Equivalent young modulus)} \tag{5}$$

A square patch test with a side length 2d and a crack length d is considered (Figure 1 a&b). The plate is composed of a graphite-epoxy material with the following orthotropic properties:

E_1 = 114.8 GPa, E_2 = 11.7 GPa, G_{12} = 9.66 GPa, υ_{12} = 0.21

To evaluate the overall error of the proposed simulation, the following error norm is used, where J_{XFEM} is the exact reference result reported by Asadpour et al. [1]:

$$err \ \% = {(J_{EFPM} - J_{XFEM})}/{J_{XFEM}} * 100 \tag{6}$$

Figure 1(c-f) depicts the mode I displacement contours over the cracked plate obtained by the proposed approach in comparison with the exact solution [1]. The total number of nodes is 740 and each cloud includes at least 12 nodes. The obtained error norm is about 20%. The numerical results have shown good agreement with available analytical solutions.

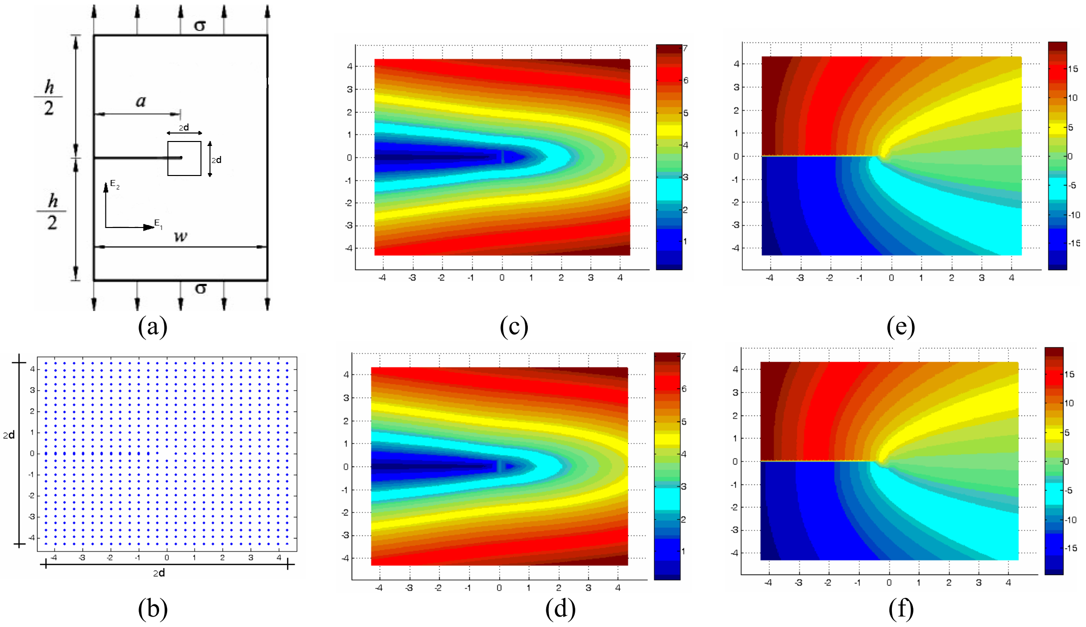

Figure 1- a) Geometry and loading of a single edge crack in a rectangular plate, b) nodal distribution, c) u_x (FPM) contours, d) u_x (exact) contours, e) u_y (FPM) contours, f) u_y (exact) contours.

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