

## Directly coupled immersed finite element method for rigid body-flow interaction problems

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### ABSTRACT

For a couple of recent decades, although computational power has been drastically increased and efficient numerical methods have developed to solve the fluid-structure interaction (FSI) problems, the accuracy still have been issues in this field. Because of its complexity, however, the accurate computation with less time is far off as usual. For instance, conventional methods solve the FSI problem through coupling and merging between fluid and solid at each time step. When the solid moves in fluid, re-meshing parts should be inevitably involved at each time step and it will be mostly a time consuming work in computation. To overcome such difficulty, immersed type of methods appear, for example, the immersed boundary (IB) method [1], the extended finite element method [2], and the fictitious domain method [3]. Especially, IB method was developed at first for the heart simulation. After that, it has been applied to various kinds of FSI problems and even to complex flows with fixed rigid boundary [4]. More recently, inspired by IB method, the immersed finite element method (IFEM) [5], the immersed continuum method (ICM) [6], and the extended immersed boundary method (EIBM) [7] were developed to efficiently solve fluid-structure interaction problems. The main advantage of IFEM is to be able to exclude the re-meshing during the simulation of moving structures in fluid, which is a common advantage with IB method. IB methods use the finite difference scheme, while the IFEM employs two kinds of finite element meshes: the structure mesh at the initial time and fluid mesh on the entire domain. Therefore, the fluid mesh is viewed as Eulerian type while the structure mesh is described as Lagrangian in conjunction with so-called flow map and Euler-Lagrangian mapping. Salient feature only in IFEM is that the reproducing kernel particle method (RKPM) is employed instead of the discrete Dirac function in IB method in order to readily distribute the FSI force onto the fluid. Thus, the immersed finite element method (IFEM) takes places as a method developed for the purpose of solving effectively fluid-structure interaction problems.

In this talk, fluid-structure interaction (FSI) problems in 2D between the incompressible Navier-Stokes flow and rigid structure are considered and we make use of the transformed finite element basis functions to replace discrete Dirac delta functions such as RKPM. This replacement makes the numerical

support of FSI force distributed to the fluid smaller than that in case where the discrete Dirac delta function is used. In the finite element formulation, reducing the support size of distributed FSI force affects the accuracy of numerical solutions near the structure [8]. The comparison of our numerical solution for particulate flows shows this fact well. We calculate particulate flows of rigid bodies and compare them with the preceding results of reliability to show a good agreement to them. In fact, this IFEM is developed based on the directly coupled FSI force treatment, which is associated with transformed finite element basis functions between fluid and structure domains in exchange for discrete Dirac delta functions. This change can be expected to remove the inaccuracy due to the FSI force distribution to the fluid using the discrete Dirac delta function. In practical point of view, the support of FSI force is concentrated exactly on the structure domain itself. The transformed finite element basis functions introduced here are able to optimally reduce the support size of distributed FSI force in the viewpoint that the support is composed of union of elements. Therefore, the emphasis in this paper is laid on achieving the optimal support in IFEM by replacing the discrete Dirac delta approximation with the transformed finite element basis functions. The immediate effect of the optimal support is expected to appear in accuracy improvement.

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