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A FRAMEWORK FOR DERIVING STABLE FINITE ELEMENT FORMULATIONS FOR INCOMPRESSIBLE VISCOUS FLOWS

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ABSTRACT

It is well known that the Galerkin method is ill suited to address convection dominated problems and that the Babuška-Brezzi condition must be satisfied when using velocity-pressure mixed formulations for incompressible viscous flows. However, as a result of many years of research in the field [1-4], a number of Petrov-Galerkin stabilised formulations that successfully overcome the above mentioned difficulties are presently available for implementation in finite element CFD codes.

In this paper we present a framework for deriving stable finite element formulations for incompressible viscous flows. Note that we use here the word *stable* instead of *stabilised* to emphasise the fact that in the present context the so-called stabilising terms emerge naturally in the derivation, rather than being introduced *a priori* in the variational formulation of the problem.

In our procedure time-discretisation using finite differences precedes the use of finite elements in space. Least-squares approximations are employed to minimise the residuals of the time-discretised governing equations with respect to the degrees of freedom. The finite element formulations arising from the proposed framework are naturally endowed with terms of SUPG type, suitable to deal with convection dominated flow, and with other stabilising terms that permit the use of equal order interpolation for velocity and pressure. Here, the stabilisation terms appear multiplied by the time-step, which permits some insights on the relationship between local time-steps and the local stabilisation parameters used in other stabilized finite element formulations. Besides, we also show that *discontinuity-capturing* terms can be introduced in the proposed framework if we adopt the concept of effective transport velocity discussed in [5].

In previous works we have applied the method to transient incompressible flows [6], to free and mixed convection problems [7], and in the simulation of flow with transport and decay of radioactive material [8]. In references [6-8] a segregated solution procedure has been employed, where pressure, velocity components and other flow transported quantities were updated sequentially within a time-step. In this paper we present our ideas in a broader perspective. We show that using different time-

discretisation schemes the proposed framework may also lead to coupled solution procedures, where all degrees of freedom are updated simultaneously during each time-step.

In an important seminal work, Hughes [9] presented the *Variational Multiscale Method*, clarifying the relationship between subgrid models and stabilized finite element formulations. Here we show that the stabilising terms introduced within the present framework lead to formulations that are equivalent to a Galerkin approximation of spatially filtered LES equations, where the subgrid models are proportional to the discretisation residuals.

Finally, some representative numerical examples involving the simulation of flow and heat transfer are presented to demonstrate the effectiveness of the present approach.

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