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Anisotropic Elasticity Tensors at Reference State– Approximations with Polyconvex Energies

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ABSTRACT

In order to simulate boundary value problems with anisotropic hyperelastic constitutive equations at moderate strains polyconvex energy densities can be used because the existence of solutions of the underlying variational functional is then automatically guaranteed. We focus therefore on polyconvex energy functions for the phenomenological description of realistic anisotropic materials. This work shows the capability to approximate experimental measurements of anisotropic materials with anisotropic fourth-order elasticity tensors at the reference state obtained by using polyconvex energies.

In finite elasticity the existence of minimizers is guaranteed if the variational functional to be minimized is sequentially weakly lower semicontinuous (s.w.l.s.) and coercive. Polyconvex functions are always s.w.l.s. and are therefore usually considered, see [1]. For isotropic materials there already exist polyconvex energy densities. Transversely isotropic and orthotropic coordinate-invariant energy functions that satisfy the polyconvexity requirement have been first proposed in [2,3]. Furthermore, a concept for the construction of polyconvex energy densities for arbitrary anisotropy is given in [4]. This method is based on the introduction of anisotropic metric tensors that reflect the symmetry properties of the associated crystal class. Since these energy functions automatically fulfill the principle of material symmetry, the polyconvexity requirement and the stress free reference configuration condition we consider them for the approximations of anisotropic fourth-order elasticity tensors at the reference state to referential data of realistic anisotropic materials. Here, the polyconvex free energy function is assumed to be of the additive type

$$\psi = \psi^{iso}(I_1, I_2, I_3) + \psi^{aniso}(I_3, J_{4j}, J_{5j}) \tag{1}$$

with the Mooney-Rivlin model as isotropic part

$$\psi^{iso} = \alpha_1\, I_1 + \alpha_2\, I_2 + \delta_1\, I_3 - (2\alpha_1 + 4\alpha_2 + 2\delta_1)\ln(\sqrt{I_3}), \quad \text{with} \quad \alpha_1,\ \alpha_2,\ \delta_1 \geq 0, \tag{2}$$

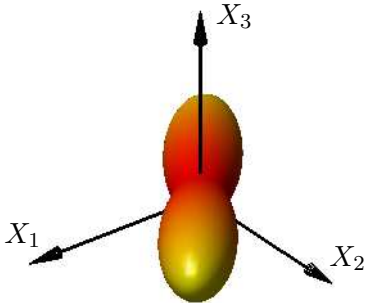
and the anisotropic part given by

$$\psi^{aniso} = \sum_{r=1}^{n}\sum_{j=1}^{m} \xi_{rj}\Big[\frac{1}{\alpha_{rj}+1}\frac{1}{(g_j)^{\alpha_{rj}}}(J_{4j})^{\alpha_{rj}+1} + \frac{1}{\beta_{rj}+1}\frac{1}{(g_j)^{\beta_{rj}}}(J_{5j})^{\beta_{rj}+1} + \frac{g_j}{\gamma_{rj}}(I_3)^{-\gamma_{rj}}\Big],$$
$$\text{with} \quad \alpha_{rj}, \beta_{rj}, \xi_{rj} \geq 0 \quad \text{and} \quad \gamma_{rj} \geq -\tfrac{1}{2}. \tag{3}$$

Here the relations $g_j := \mathrm{tr}\boldsymbol{G}_j,\ J_{4j} = \mathrm{tr}[\boldsymbol{C}\boldsymbol{G}_j],\ J_5 = \mathrm{tr}[\mathrm{Cof}[\boldsymbol{C}]\boldsymbol{G}_j]$ are considered and m-different metrics $\boldsymbol{G}_j$ are used. Since ψ^{aniso} like ψ^{iso} is coercive itself we are able to neglect the isotropic part, see [4]. The fitting of the fourth-order elasticity tensor near the natural state is done by minimizing the error function

$$e = \frac{\| \mathbb{C}^{(V)comp} - \mathbb{C}^{(V)exp} \|}{\| \mathbb{C}^{(V)exp} \|} , \tag{4}$$

where $\mathbb{C}^{(V)comp} \in \mathbb{R}^{6\times 6}$ denotes the computed tangent moduli at the reference state, i.e., $\mathbb{C}^{(V)comp} := 4\partial_{\boldsymbol{C}\boldsymbol{C}}\psi|_{\boldsymbol{C}=\boldsymbol{1}}$, and $\mathbb{C}^{(V)exp} \in \mathbb{R}^{6\times 6}$ designates the experimentally determined elasticity tensor in Voigt notation. As an example of the fitting described above, we show the results of the approximation for the monoclinic material Augite with the proposed model by only considering the anisotropic energy term.



$$\mathbb{C}^{(V)exp} = \begin{bmatrix} 217.8 & 72.4 & 33.9 & 24.6 & 0 & 0 \\ & 181.6 & 73.4 & 19.9 & 0 & 0 \\ & & 150.7 & 16.6 & 0 & 0 \\ & & & 51.1 & 0 & 0 \\ & sym. & & & 55.8 & 4.3 \\ & & & & & 69.7 \end{bmatrix}$$

Figure 1: Monoclinic Augite: characteristic surface of Bulk moduli, elasticities [GPa].

For Augite we obtain with $n = m = 3$ and the monoclinic metric tensors

$$\boldsymbol{G}_1^m = \begin{bmatrix} 2.301 & 0.778 & 0 \\ 0.778 & 0.559 & 0 \\ 0 & 0 & 1.500 \end{bmatrix},\ \boldsymbol{G}_2^m = \begin{bmatrix} 1.707 & -0.123 & 0 \\ -0.123 & 0.009 & 0 \\ 0 & 0 & 0.245 \end{bmatrix},\ \boldsymbol{G}_3^m = \begin{bmatrix} 1.096 & -0.607 & 0 \\ -0.607 & 1.191 & 0 \\ 0 & 0 & 2.067 \end{bmatrix}$$

a relative error $e \approx 3.6\%$.

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