8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Convergence results for the Bayesian inversion theory

Andreas Neubauer[1] and *Hanna K. Pikkarainen[2]

[1] Industrial Mathematics Institute
Johannes Kepler University Linz
Altenbergerstrasse 69, A-4040 Linz, Austria
neubauer@indmath.uni-linz.ac.at
fox.indmath.uni-linz.ac.at/wwwpriv/

[2] Institute of Mathematics
Helsinki University of Technology
P.O. Box 1100, FI-02015 TKK, Finland
hanna.pikkarainen@tkk.fi
math.tkk.fi/people/hanna.pikkarainen.en.html

Key Words: *Bayesian inversion theory, Convergence rates, Parameter choice rules.*

ABSTRACT

In this talk we derive convergence results for regularized solutions of linear inverse problems obtained by the Bayesian approach in the Ky Fan metric. We show that the convergence rate is order optimal in finite dimensional spaces. Moreover, we prove that order optimal rates can be obtained for weighted Bayesian solutions when the dimension goes to infinity.

We study the solution of the linear ill-posed problem

$$Tx = y \tag{1}$$

from noisy measurements of y, where T is a bounded linear operator between real separable Hilbert spaces $\mathcal{X}$ and $\mathcal{Y}$. We are interested in the case where the noise in the measurements can be modelled by a Gaussian random variable.

In a first step, we treat a finite dimensional version of equation (1) as it occurs when this problem is discretized, i.e., we deal with the solution of the problem

$$A\bar{x} = \bar{y}\,, \tag{2}$$

where $A \in R^{m\times n}$ is a (usually ill-conditioned) matrix, $\bar{x} \in R^n$, and $\bar{y} \in R^m$. We use the Bayesian inversion theory for obtaining an approximate solution of (2). We show that both the conditional mean estimate and the posterior distribution obtain convergence rates with the same order as the noise when measured in the Ky Fan metric under the assumption that the prior and the noise distributions are normal and independent.

Furthermore, we study convergence issues when problem (1) is discretized via projection. We show that the finite dimensional conditional mean estimate cannot converge in the Ky Fan metric to the least squares minimum norm solution as the dimension tends to infinity. We propose a weighted Bayesian approach which can be obtained by changing the norm in the underlying space $\mathcal{X}$. We prove that both the weighted conditional mean estimate and the weighted posterior distribution obtain order optimal convergence rates when measured in the Ky Fan metric.

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