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OPTIMAL ORTHOTROPIC MATERIAL ORIENTATION BY THE USE OF POLAR REPRESENTATION

*** A. Vincenti[1], B. Desmorat[2,3]**

[1] IJLRDA, Univ. Paris 6
"Pierre et Marie Curie"
Case 161, 4 place jussieu,
75252 Cedex, Paris, France
vincenti@ccr.jussieu.fr

[2] IJLRDA,Univ. Paris 6
"Pierre et Marie Curie"
Case 161, 4 place jussieu,
75252 Cedex, Paris, France
borisdes@ccr.jussieu.fr

[3] Univ. Paris-Sud 11
91405, Orsay, France Cedex,
Paris, France

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ABSTRACT

Structural optimisation means a process for the design of structures showing the best performance according to given criteria. The application of variational principles in this field leads to topology optimisation, i.e. the definition of the optimal distribution of matter within a given domain and under given loads, in order to achieve maximum stiffness. When applied to composite structures, the optimisation process can even take into account the architecture of the constitutive materials, which are fiber reinforced composites, suitable to be optimally designed [1]. As a matter of fact, composites are themselves structured materials, and depending on the architecture of their reinforcements, they can show rather complex anisotropic properties and multiple couplings. Nevertheless, one of the main difficulties in the optimal design of composite structures is related to the link between the local material architecture (e.g. the layup of a composite laminate) and the global response of the structure.

For a given state of load applied to the structure and according to a variational principle, the anisotropic directions of a composite material can be optimally oriented and their elastic properties can be optimally tailored in order to maximise the global stiffness of the structure. Few authors already dealt with the problem of minimisation of elastic energy with respect to material orthotropy orientation for a two-dimensional orthotropic elastic medium under a given state of stress: Sacchi-Landriani and Rovati [2] used the cartesian representation and they obtained a partial answer to the problem, whilst Pedersen et al., [3] and [4], completely solved the problem by the use of the lamination parameters. Nevertheless, the use of lamination parameters leads to rather cumbersome expressions.

In this paper, we propose the application of the polar representation method of plane anisotropy [5] to the minimisation of elastic energy. After giving the expression of the elastic energy in terms of polar parameters of stress and of material compliance, we show how the use of polar invariants allows a simple and clear discussion for the minimisation problem with respect to the orthotropy orientation of the material. Finally, we give the expression of the minimum of elastic energy in terms of polar parameters of stress and of material compliance.

This work opens the way to a very general approach to the topology optimisation of two-dimensional composite structures by the use of the polar representation.

Sensitivity analysis Given a two-dimensional elastic medium, under a given state of stress represented in terms of its polar components T and R, the expression of the complementary energy is :

$$W_c = 2t_0 R^2 + 4t_1 T^2 + 2(-1)^k r_0 R^2 \cos 4\gamma + 8r_1 T R \cos 2\gamma \tag{1}$$

where t_0, t_1, r_0 and r_1 are polar components for material compliance, and angle $\gamma = \varphi_1 - \varPhi$ is the difference between φ_1, angle of the principal orthotropy axes for compliance, and $\varPhi$, angle of the first principal stress direction. Finally, the polar parameter k can only take values 0 or 1, and it is related to the orthotropy shape for the material compliance [5]. Conditions of stationarity for W_c with respect to angle φ_1 are the following:

$$\frac{\partial W_c}{\partial \varphi_1} = 0 \qquad \Leftrightarrow \begin{cases} r_0 = r_1 = 0 \text{ (case of isotropic material)} \\ R = 0 \text{ (case of spherical stress tensor)} \\ \sin(2\gamma) = 0 \Rightarrow \gamma = 0 \text{ or } \gamma = \pi/2 \\ \cos(2\gamma) = -\zeta, \text{ if: } -1 \le \zeta \le 1 \ \left(\text{with } \zeta = \frac{r_1 T}{(-1)^k r_0 R}\right) \end{cases} \tag{2}$$

Depending on the values of the polar parameters of stress and of material compliance (parameter $|\zeta|$ and parameter k), the stationary points can be either relative minima or maxima, and the results of the minimisation problem are shown in Table 1 (σ_I and σ_{II} are the principal stresses).

	$k = 1$	$k = 0$
$\lvert\zeta\rvert \ge 1$	$\varphi_1 = \text{dir}[\min(\lvert\sigma_I\rvert, \lvert\sigma_{II}\rvert)]$	$\varphi_1 = \text{dir}[\min(\lvert\sigma_I\rvert, \lvert\sigma_{II}\rvert)]$
$\lvert\zeta\rvert \le 1$	$\varphi_1 = \text{dir}[\min(\lvert\sigma_I\rvert, \lvert\sigma_{II}\rvert)]$	$\varphi_1 = \text{dir}[\min(\lvert\sigma_I\rvert, \lvert\sigma_{II}\rvert)] \pm \frac{1}{2}\arccos\zeta$

Table 1: Optimization results, conditions for the minimisation of complementary energy W_c

We give therefore the expressions of the minimum of W_c with respect to material orientation for the different cases shown in Table 1:

- case $k = 1$: ${W_c}^{min} = 2t_0 R^2 + 4t_1 T^2 - 2r_0 R^2 - 8r_1 |T| R$
- case $k = 0$ and $|\zeta| \ge 1$: ${W_c}^{min} = 2t_0 R^2 + 4t_1 T^2 - 2r_0 R^2 - 8r_1 |T| R$
- case $k = 0$ and $|\zeta| \le 1$: ${W_c}^{min} = 2t_0 R^2 + 4t_1 T^2 - 2r_0 R^2 - 4\frac{r_1^2}{r_0}|T|^2$

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