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Effect of base flow uncertainty on Couette flow stability

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ABSTRACT

The eigenvalues of the Orr-Sommerfeld operator determine the stability of exponentially growing disturbances in parallel and quasi-parallel flows such as Couette flows [3]. This work investigates the sensitivity of these eigenvalues to modifications of the base flow. Our approach is to regard these modifications as random quantities. Such base flow uncertainties may represent differences between the flow in experimental conditions and its ideal numerical counterpart. This study reveals that those base flow uncertainties can be destabilizing, although the linear stability theory (LST) predicts the Couette flow to be unconditionally for all Reynolds numbers [3]. In fact, turbulence occurs in experiments at Reynolds number as low as 300 [4]. Several authors have examined the possible causes of this disagreement. Using a variational technique, Bottaro *et. al* have shown that very small base flow variations can displace the eigenvalues towards the unstable region of the imaginary plane [1]. Schmid [2] examined the response of the optimal energy growth of a profile subject to four localized random perturbations with Gaussian peak magnitudes. However, the stochastic moments obtained from Monte Carlo simulations showed that, despite the variation in the energy growth, the flow did not become unstable.

In this work, we use a Karhunen–Loeve (KL) spectral expansion to treat the base flow stochastic variation as a Gaussian random process. We consider the random process to exhibit some degree of correlation in the cross-stream direction. We use a Gaussian covariance kernel with correlation length Cl to represent this correlation. The orthogonal basis used in the KL decomposition are Hermite polynomials. The chosen number of terms N in the expansion is dictated by the decay of the covariance kernel eigenvalues. Examples of base flow realizations obtained from the KL expansion are shown in Figure 1. Here, we choose a typical correlation length of $Cl = 1$ and we use $N = 8$ terms in the expansion. A deterministic LST spectral solver is then coupled to a non-intrusive stochastic collocation solver to propagate the base flow uncertainties through the system. The non-intrusive approach does not require any substantial modifications to the deterministic solver. The evaluation of the solution moments is equivalent to computing multi-dimensional integrals over the probability domain. Different ways of dealing with high-dimensional integrations can be considered depending on the prevalence of accuracy versus efficiency. Here, we use a numerical quadrature of Gauss-type by full tensor products. This approach is very accurate and remains computationally efficient for a moderate number of random dimensions. Different sparse quadrature techniques are also investigated in order to speed–up the process.

Table 1 lists the mean values μ and coefficients of variation $C_v = \mu/\sigma$ of the most unstable eigenvalue for different correlation lengths. The mean value decreases for increasing Cl which means that it gets closer to the instable region for small Cl. We notice that Cl stops affecting the mean value for $Cl = 4$ which corresponds to twice the channel width. The coefficient of variation keeps decreasing for increasing Cl due to the change in variance. Figure 2 shows the variance of the first forty eigenvalues (sorted by imaginary part). We observe that the variance increases for increasing Cl. Moreover, we have

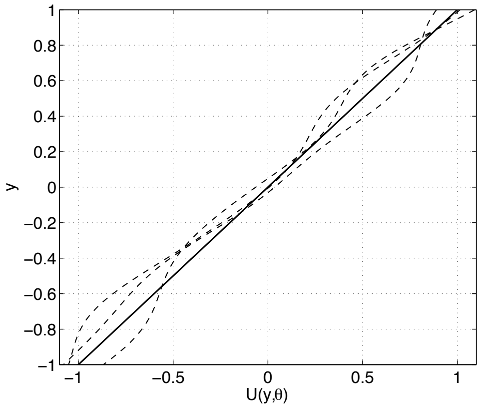

Figure 1: Comparison between ideal (solid) and randomly perturbed (dashed) Couette base flow velocity profiles.

	Cl = 1	Cl = 2	Cl = 4
μ	-0.09345	-0.20528	-0.20863
C_v	0.70825	0.05971	0.02571

Table 1: Mean value and coefficients of variation of the most unstable eigenvalue for different C_l.

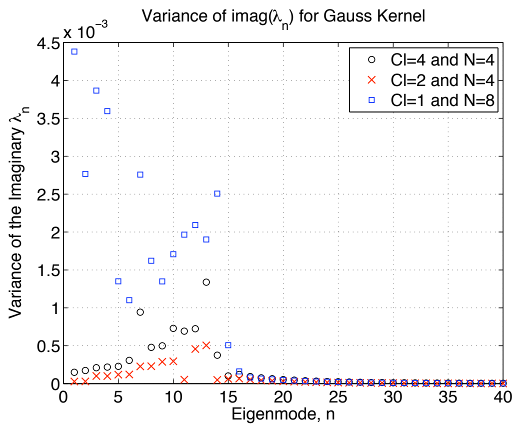

Figure 2: Variance of the first forty eigenvalues, sorted by imaginary part, of the Orr-Sommerfeld operator for different stochastic perturbation correlation lengths.

found qualitative agreement with the existing literature that shows that the eigenvalues located at the intersection of different branches are the most sensitive to changes. In addition to those results we will also present the response of the transient energy growth and probability density functions of pertinent quantities and introduce the notion of the most *probable* solution.

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