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An Analytical / Numerical Approach Based on Integral Equations for Crack Problems of Gradient Elasticity

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ABSTRACT

In the present study, an analytical / numerical approach is introduced for the solution of crack problems of gradient elasticity. This approach is based on hypersingular integral equations with a cubic singularity. A new mechanical quadrature is constructed for the numerical solution of a pertinent system of these equations. The boundary value problems of mode I and mode II cracks were first attacked with the Fourier transform. Then, a system of coupled hypersingular integral equations for each case was formulated. It is shown that our approach is capable in providing a very detailed full-field solution to crack problems of gradient elasticity.

The theory of gradient elasticity is used here to model the mechanical response of solids with microstructure. This theory is perhaps the most popular one, in recent years, among generalized continuum theories. We employ a simple but yet rigorous version of the general dipolar gradient theories of Toupin [1] and Mindlin [2]. This version involves an isotropic linear response and only one material constant (the so-called gradient coefficient) additional to the standard Lame constants [3], [4]. The strain-energy density function assumed, besides its dependence upon the standard strain terms, depends also on strain gradients.

The results for the near-tip fields show significant departure from the predictions of the classical fracture mechanics. In view of these results, it seems that it is inadequate to analyze crack problems in microstructured materials employing classical fracture mechanics. Indeed, the present results indicate that the stress distribution ahead of the tip exhibits a local maximum that is bounded. This maximum value may serve as a measure of the critical stress level at which further advancement of the crack may occur. Also, in the vicinity of the crack tip, the crack-face displacement closes more smoothly as compared to the classical analysis. The latter result of our analysis can be explained physically since materials with microstructure behave in a more rigid way (having

increased stiffness) as compared to materials without microstructure (i.e. materials governed by classical continuum mechanics). Some aspects of the behavior of the mode I and II solutions are also shared by the solution behavior of the mode III case treated earlier by the first author [3].

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