8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

MASS CONSERVATIVE APPROXIMATION OF FLOW IN POROUS MEDIA WITH APPLICATIONS TO ENVIRONMENTAL STUDIES

*** Markus Bause**[1]

[1] Department Mathematics
University of Erlangen-Nuremberg
Martensstr. 3, 91058 Erlangen
Germany
bause@am.uni-erlangen.de
www1.am.uni-erlangen.de/˜bause

Key Words: *Subsurface Flow, Multicomponent Reactive Transport, Mixed Finite Element Method, Multi Point Flux Approximation.*

ABSTRACT

Accurate and reliable simulations of moisture fluxes through porous media are desirable in many areas, in particular, in hydrological and environmental engineering and for industrial oil exploration studies. In the simulation of subsurface flow, the geology which includes composite soil formations, that may be intermittently saturated and drained, and non-orthogonal domains in the media is a major challenge. In environmental studies, the prediction of the flow field is usually of greater importance than the approximation of the pressure head itself, since the flux is responsible for the availability of chemical species in simultaneous transport processes. In this contribution various numerical and theoretical aspects of a reliable and efficient approximation of coupled subsurface flow and transport phenomena are adressed.

The mixed finite element method (cf. [2, 4, 5]) and the multi point flux approximation method (cf. [1]) have shown to be suitable for the numerical simulation of such flows in porous media. Both schemes are locally mass conservative and provide a flux approximation as part of the formulation. In this contribution the application of the mixed finite element method with lowest order Raviart-Thomas ($\mathbf{RT}_0$) and Brezzi-Douglas-Marini ($\mathbf{BDM}_1$) elements and of the multi point flux approximation (**MPFA**) method to saturated and unsaturated flow in a composite and heterogeneous porous medium is studied and the accuracy of the pressure and flux approximation in various norms is analyzed. Advantage of the BDM_1 approach over the RT_0 and MPFA method is its ability to be able to yield a higher second order accurate approximation of the flux. For the numerical approximation of the accompanying multicomponent reactive transport process a higher order conforming finite element approach is used which has shown to be less numerically diffusive than lowest order methods; cf. [3]. The efficiency of the techniques is illustrated for some real-world field scale subsurface flow and reactive transport scenarios. Extensions to two phase flow with application to remediation techniques and CO_2 sequestration are also given.

As a prototype for the pressure equation in a reservoir simulation, single phase Darcy flow,

$$-\nabla \cdot (\boldsymbol{K}(\boldsymbol{x})\nabla p) = f \text{ in } \Omega \,, \quad p = g \text{ on } \Omega \,, \tag{1}$$

ist studied. Here, Ω is a bounded two-dimensional domain, p denotes the pressure, $\boldsymbol{K}$ the permeability and $\boldsymbol{u} = -\boldsymbol{K}\nabla p$ the Darcy velocity. To describe the numerical methods, let $\mathcal{T}_h = \{T\}$ be a finite element decomposition of mesh size h of the polyhedral domain $\overline{\Omega}$ into closed triangles T. We form discrete subspaces W_h of $L^2(\Omega)$ and $\boldsymbol{V}_h$ of $\boldsymbol{H}(\Omega; \text{div})$. For instance, let $\boldsymbol{BDM}_1(T) = (P_1(T))^d$ denote the lowest order Brezzi–Douglas–Marini space where $P_i(T)$ is the set of all polynomials on T of degree less or equal than i. We set $W_h = \{w_h \in L^2(\Omega) \mid w_{h|T} \in P_0(T)\ \forall T \in \mathcal{T}_h\}$ and $\boldsymbol{V}_h = \{\boldsymbol{v}_h \in \boldsymbol{H}(\Omega; \text{div}) \mid \boldsymbol{v}_{h|T} \in \boldsymbol{BDM}_1(T)\ \forall T \in \mathcal{T}_h\}$. A BDM_1 mixed approximation $\{p_h, \boldsymbol{u}_h\}$ of (1) then reads as: *Find* $\{p_h, \boldsymbol{u}_h\} \in W_h \times \boldsymbol{V}_h$ *such that*

$$\langle \nabla \cdot \boldsymbol{u}_h, w_h \rangle = \langle f, w_h \rangle, \quad \langle \boldsymbol{K}^{-1}\boldsymbol{u}_h, \boldsymbol{v}_h \rangle - \langle p_h, \nabla \cdot \boldsymbol{v}_h \rangle = -\langle g, \boldsymbol{v}_h \cdot \boldsymbol{\nu} \rangle_{\partial\Omega} \tag{2}$$

holds for all $\{w_h, \boldsymbol{v}_h\} \in W_h \times \boldsymbol{V}_h$.

Multi point flux approximation control volume methods (cf. [1]) are discretization techniques developed by the oil industry for accurate and reliable reservoir simulation. In this approach more than two pressure values are used in the flux approximation. The basic idea of MPFA is to divide each cell into subcells and then to assume linear variation of the pressure in each subcell. The half edge flux is determined by Darcy's law on the linear pressure variation in each subcell. The method is defined by assuming continuous fluxes across each half cell edge, belonging to a subcell, and a continuity condition at one point for the pressure across each half edge. Multi point flux approximations can offer explicit discrete fluxes, which is not possible to get from mixed finite element methods and is in particular advantageous for simulating multi phase flow. We apply the MPFA O-method on triangular grids. A new error estimate for this approach is given.

Exemplarily, Fig. 1 shows the calculated errors of the flux approximation for a sequence of sucessively refined and stochastically distorted meshes.

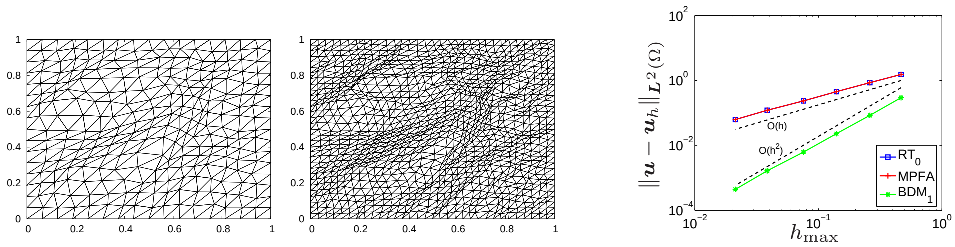

Figure 1: Flux approximation on stochastically distorted, non-orthogonal grids.

REFERENCES

[1] I. Aavatsmark et al. "Discretization on unstructured grids for inhomogeneous, anisotropic media. Part I and II". *SIAM J. Sci. Comput.*, Vol. **19**, 1700–1736, 1998.

[2] M. Bause. "Higher and lowest order mixed finite element approximation of subsurface flow problems with solutions of low regularity". *Adv. Water Resour.*, Vol. **31**, 370–382, 2008.

[3] M. Bause and P. Knabner. "Numerical simulation of contaminant biodegradation by higher order methods and adaptive time stepping". *Comp. Vis. Sci.*, Vol. **7**, 61–78, 2004.

[4] M. Bause and P. Knabner. "Computation of variably saturated subsurface flow by adaptive mixed hybrid finite element methods". *Adv. Water Resour.*, Vol. **27**, 565–581, 2004.

[5] M. Bause and J. Hoffmann. "Numerical study of mixed finite element and multi point flux approximation of flow in porous media". *Proceedings of ENUMATH 2007, Graz (Austria)*, submitted, 1–8, 2007.