

PARALLELIZATION OF THE SPATIAL OPERATOR ALGEBRA FOR DYNAMICS OF MULTIBODY SYSTEMS

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ABSTRACT

The Spatial Operator Algebra (SOA) [1] methods for generating and solving the equations of motion of multibody systems, as developed by Rodriguez, Jain and others at NASA Jet Propulsion Laboratory, have been extensively and successfully used at NASA in modelling the dynamics of various space and planetary vehicles. Various researchers [2,3] have also successfully used these methods for modelling the dynamics of bio-molecular systems.

With a growing trend towards modelling larger and more complex systems, there is always a need for faster simulation turn around times. This can be improved in two ways (i) by improving the computational efficiency of the underlying algorithms, and (ii) by using parallel computational resources.

The SOA belongs to the family of methods now commonly known as recursive state-space $O(n)$ methods. Similar to the other methods of this family, the SOA methods achieve the theoretically maximum computational efficiency in serial implementation i.e. computational costs scale linearly with the number of degrees of freedom in the system. Consequently, in serial implementation, the opportunity for increase in computational efficiency is very limited. Further improvements can only be achieved by exploiting parallel computing resources. Unfortunately, due to their recursive nature, the SOA methods do not scale well in parallel implementation. These algorithms are based on order dependent, sequential operations and hence considerably reduce the scope for concurrent processing.

In this work, a method is presented for improving the computational efficiency of the SOA methods in parallel implementations. This method is based on a hybrid coupling of the traditional SOA methods with the Divide and Conquer Approach (DCA) [4,5]. The Divide and Conquer method is based on the idea of decomposing a large problem into smaller sub-problems, which are solved concurrently, and then combining the solutions of the sub-problems to form the solution of the larger global problem. Similarly, the fundamental idea of this work is the decomposition of the system to be modelled into smaller subsystems. The equations of motion of these subsystems are formulated concurrently using the efficient SOA methods. The equations of these

subsystems are then coupled together in an order independent manner using a DCA based method. Using the DCA, the boundary conditions of the subsystems are calculated using an efficient binary tree approach. Finally, based on the boundary conditions, the equations of motion of the subsystems are solved concurrently using the efficient SOA methods.

This presentation will briefly introduce the underlying formulations of the SOA and the DCA, and then focus on the analytical development of the new hybrid method. The application of the method for generating and solving the equations of motion of multi-rigid body systems in chain, tree and closed kinematic loop configurations (constrained systems) will be demonstrated. The ability of the method to overcome traditional rank deficiency issues for constrained systems will be highlighted. Extension of the method for multi-flexible body systems modelled under the small deformation large displacement approximation will also be demonstrated.

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