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Effects of Topography on Rotational Ground Motions under Incident Elastic Waves Using Boundary Methods

by

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ABSTRACT

The surface displacement fields at the surface of topographical profiles under incidence of elastic plane waves (Rayleigh, P, SV and SH) can be computed using both the Indirect Boundary Element Method (IBEM) and the Method of Fundamental Solutions (MFS). In these approaches the diffracted or scattered field is constructed by means of discretized integral representations in terms of the Green function (which is the displacement at a given location due to a concentrated unit load in other point) and force densities (for IBEM) or forces (for MFS). Once such force densities or forces are obtained, the ground motion can be computed. The computation of ground motion rotations implies the application of the rotational operator to the displacement field. This is done using either numerical derivatives or analytical expressions to compute the rotational Green tensor. The boundary integral computation requires dealing explicitly with the singularity of Green function.

The 2D Green function is given by $G_{ij}(x,\xi) = (i/8\rho)\{(A+B)\delta_{ij} - 2B\gamma_i\gamma_j\}$. Where γ_i = direction cosine for direction i of vector from ξ to x, δ_{ij} = Kronecker delta, we also have that $A = H_0^{(2)}(qr)/\alpha^2 + H_0^{(2)}(kr)/\beta^2$ and $B = H_2^{(2)}(qr)/\alpha^2 - H_2^{(2)}(kr)/\beta^2$, with α , β = wave propagation velocities of P and S waves, respectively, r = distance between points ξ_l and x; $H_0^{(2)}, H_2^{(2)}$ = Hankel functions of the second kind and orders 0 and 2, respectively, $q = \dfrac{\omega}{\alpha}$ and $k = \dfrac{\omega}{\beta}$ are the wave numbers and ω = angular frequency in rad/sec.

Applying ½ the rotational operator to the Green function components we can easily obtain $\Phi_{21} = \frac{1}{2} rot(G_{i1}) = \frac{1}{2}\left(\frac{\partial G_{11}}{\partial x_3} - \frac{\partial G_{31}}{\partial x_1}\right)$ and $\Phi_{23} = \frac{1}{2} rot(G_{i3}) = \frac{1}{2}\left(\frac{\partial G_{13}}{\partial x_3} - \frac{\partial G_{33}}{\partial x_1}\right)$. Here the first index indicates rotation while the second indicates force directions, respectively. The Green function derivative is given by

$$\frac{\partial G_{ij}}{\partial x_k} = \frac{i}{4\rho}\frac{1}{r}\left[\left(\frac{D(qr)}{\alpha^2} - \frac{D(kr)}{\beta^2}\right)\gamma_i\gamma_j\gamma_k + \frac{D(kr)}{\beta^2}\delta_{ij}\gamma_k - 4B\gamma_i\gamma_j\gamma_k + B\left(\delta_{ij}\gamma_k + \delta_{ik}\gamma_j + \delta_{jk}\gamma_i\right)\right],$$

where $D(z) = zH_1^{(2)}(z)$. After some algebra we have $\Phi_{21} = (i/8\mu)kH_1^{(2)}(kr)\gamma_3$ and $\Phi_{23} = -(i/8\mu)kH_1^{(2)}(kr)\gamma_1$.

The inplane case (SH) is simpler. The Green function is $G_{22} = (1/i4\mu)H_0^{(2)}(kr)$ and the rotations are $\Phi_{12} = \frac{1}{2} rot(G_{i2}) = -\frac{1}{2}\left(\frac{\partial G_{22}}{\partial x_3}\right)$ and $\Phi_{32} = \frac{1}{2} rot(G_{i2}) = \frac{1}{2}\left(\frac{\partial G_{22}}{\partial x_1}\right)$. After some simplification we obtain $\Phi_{12} = -\frac{i}{8\mu}kH_1^{(2)}(kr)\gamma_3$ and $\Phi_{32} = \frac{i}{8\mu}kH_1^{(2)}(kr)\gamma_1$.

These analytical expressions in 2D are essentially the same for both the in-plane and the anti-plane cases. They display an anti-symmetric behavior that also appears in the corresponding expressions in the full 3D case. The contribution of Green function singularity is identified and the IBEM set of expressions to compute displacements, tractions and rotations are given.

On the other hand, the MFS is discussed. The main advantages of this meshless method are pointed out. Here we introduce a Gaussian MFS in which instead of plain collocation we actually perform Gaussian collocation which in practice increase accuracy. One great advantage of MFS is that singularities are avoided. An analysis of the numerical performance of these two approaches is presented. The obtained results are validated using the exact analytical solution for two triangular wedge-like mountains with internal angles of 120º and 90º [1]. Thus, displacements and rotations obtained using these analytical solutions are compared with those from IBEM and MFS.

Rotations are computed for different topographical profiles and various incoming fields and concentrated sources as well in both frequency and time domains. The effects of topography on rotational ground motion are discussed with emphasis on structural response.

REFERENCE

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