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A STABILIZED DISCONTINUOUS GALERKIN FORMULATION FOR HELMHOLTZ PROBLEMS

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ABSTRACT

The Helmholtz equation belongs to the classical equations of mathematical physics that are well understood from a mathematical view point. However, the numerical approximation of the solution of those equation is still a challenging problem, despite the tremendous progress made during the last years. Indeed, the classical finite element method – based on polynomial approximation – becomes too expensive for large wavenumbers, when trying to control the numerical dispersion errors. Thus, other techniques were proposed, based on the common idea of incorporating some *a priori* knowledge about the solution. Mainly, the idea of using plane waves was developed in different frameworks, continuous and discontinuous. One may cite the partition of unity method [1], the residual-free bubbles [2] and the discontinuous Galerkin methods [3].

We propose here a new discontinuous Galerkin formulation, based on a local approximation of the solution by plane waves that satisfy the wave equation. In order to enforce a weak continuity across the element interfaces, we introduce Lagrange multipliers. The method is built in a variational formulation framework that leads to a linear system associated with a ***positive*** definite Hermitian matrix. This matrix results from using a stabilized-like technique. Therefore, we use a preconditioned conjugate gradient algorithm to solve the system without computing the resulting matrix.

Numerical results to illustrate the potential of the method for solving efficiently Helmholtz problems in the mid- and high-frequency regime will be presented. Such results are compared to the ones obtained by the discontinuous Galerkin method designed by C. Farhat *et al* in [3] and analyzed mathematically in [4].

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