

## Numerical Simulation of Cerebrospinal Fluid Flow by Fictitious Domain Method

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### ABSTRACT

This study addresses the flow of cerebrospinal fluid (CSF). The cerebrospinal fluid fills the cerebrospinal cavity so that the central nervous system is protected safely from shock or vibration. Many blood vessels are distributed throughout the brain. Therefore, its pulsation is synchronized with the heartbeat. Thereby, the CSF also has a pulsate flow.

The field of brain surgery requires knowledge of CSF flows because that flow behavior seems to affect several diseases of the central nervous system. Anatomical data of the cerebrospinal cavity can be obtained using Magnetic Resonance Imaging (MRI). The cross-sectional flow velocity of CSF can also be obtained through MRI.

This study is intended to construct a numerical model to elucidate CSF flow behaviour. The fictitious domain method<sup>4</sup> is adopted for representing brain and spinal canal geometry because that geometry is very complicated. In this study, the velocity profile on a cross-section of cerebrospinal cavity is given by MRI measurement as a boundary condition for the numerical model. We apply a friction-type boundary condition<sup>2</sup> to the boundary surrounding the spinal cord in order to represent CSF absorption and emission.

Flows of CSF are assumed to be governed by incompressible viscous Navier-Stokes equations. The original problem is defined in a time-dependent domain because of shape changes of spine and brain. We reformulate the problem by fictitious domain method into one defined in a time-independent domain by using characteristic functions.

The numerical model is discretized using finite difference method. All spatial derivatives, except for the convection terms, are approximated by the second-order central differences. The convection term is approximated by the third-order upwind scheme.

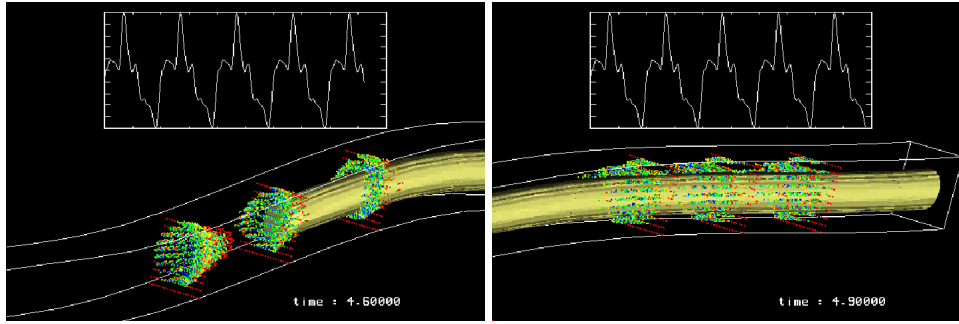


Figure 1: Particle trace around a spinal cord

Figures 1 and 2 show some results for flows around spine. Figure 1 shows flows using particle trace technique around the spinal cord near lumbar and cervical vertebrae. Figure 2 shows a flow field around the cervical spine: the absorption and emission effects are included through the friction-type boundary condition.

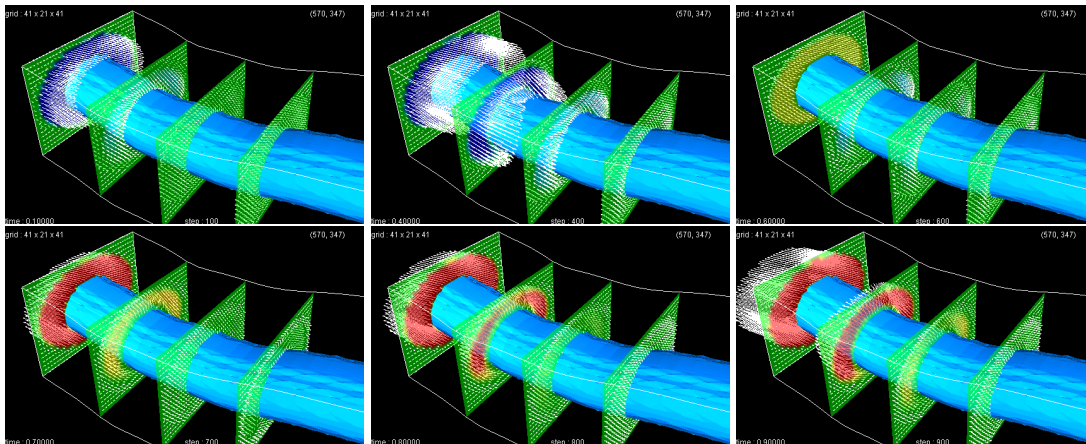


Figure 2: Absorption and emission near cervical spine

This model can be refined by comparison between numerical results and another cross-sectional velocity profiles other than the profile used here as the in-flow boundary condition. Furthermore, this model can be applied to a flow around a brain.

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