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Adaptive stabilization for a discontinuous Galerkin method for nonlinear elasticity

Adrian Lew[1], Alex Ten Eyck[1], Fatih Celiker[2]

[1] Stanford University
496 Lomita Mall
Stanford, CA 94305-4040
{lewa,alabute}@stanford.edu

[2] Wayne State University
Detroit, MI 48202
celiker@math.wayne.edu

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ABSTRACT

A difficulty encountered when formulating discontinuous Galerkin methods for nonlinear problems lies in the crafting of the stabilization strategy. A majority of current stabilization mechanisms rely on some form of penalization of the discontinuities across element boundaries, with the introduction of stabilization parameters that control their size. The choice of these parameters is always a delicate task. Values that are too small may render the method unstable. In contrast, values that are too large may only allow for very small discontinuities, returning a solution that is nearly identical to that of the underlying conforming method, but with a much larger computational cost.

In this talk we first present a set of numerical experiments showcasing the effect of the stabilization term on the accuracy of the numerical solutions in nonlinear elasticity. We then introduce two different strategies for the the general linearized elasticity problem. The first one consists in a nonstandard stabilization term, but that enables us to obtain sharper analytical lower bounds for the values of the stabilization parameters that will render the method stable. Numerical examples, however, show that the resulting lower bounds are not sharp enough: it is possible to obtain more accurate solutions with even smaller values of the stabilization parameters, while retaining the stability of the method.

The second strategy consists of a simpler stabilization term, which changes from element to element based on the lowest negative eigenvalue of the elastic moduli therein. Numerical examples in two and three dimensions are used to study the role of the stabilization parameters in the accuracy of the resulting solutions, and to compare it with a more traditional stabilization strategy. These example showcase the improved performance of the former over the latter.

We extend the two strategies to the nonlinear elastic case via an incremental variational principle. Since information about the elastic moduli is used to construct the stabilization terms, these change, or adapt, to the solution sought. Hence the name of adaptive stabilization. Numerical examples demonstrate the performance of the resulting method.