8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

AN ADAPTIVE GRADIENT SMOOTHING METHOD (GSM) FOR FLUID DYNAMICS PROBLEMS

***George XU[1], and G.R. LIU[2,3]**

[1] Institute of High Performance Computing
1 Singapore Science Park Road,
#01-01 The Capricorn,
Singapore Science Park II,
Singapore 117528
xuxg@ihpc.a-star.edu.sg

[2]Centre for Advanced Computations in Engineering Science,
Department of Mechanical Engineering,
National University of Singapore,
10 Kent Ridge Crescent, Singapore 119260
[3]Singapore-MIT Alliance (SMA), E4-04-10,
4 Engineering Drive 3, Singapore, 117576
mpeliugr@nus.edu.sg

KEY WORDS: numerical method, FVM, adaptive, fluid dynamics, GSM, GSD, Stencil.

ABSTRACT

A novel and efficient numerical method called gradient smoothing method (GSM) based on strong form of governing equations has been recently developed by Liu and Xu (2007). In GSM, the PDEs are directly discretized at nodes, which can be irregularly distributed in the physical domain, using gradient smoothing techniques. The GSM can be applied very effectively with excellent stability for arbitrary geometries, as demonstrated in Liu and Xu (2007). In this paper, our continuous efforts in the development of an adaptive GSM are presented. The objectives are to extend the GSM to resolve the flows with abrupt changes in features, such as a shock flow. Besides, a stable adaptive process is expected to be established.

In the GSM, derivatives at various locations, including nodes, centroids of cells and midpoints of cell-edges, are approximated over relevant gradient smoothing domains using gradient smoothing operation. The gradients of a field variable U at a point of interest at $\mathbf{x}_i$ in domain Ω_i can be approximated in the form of [Lucy (1977); Liu and Liu (2003)]

$$\nabla U_i \equiv \nabla U(\mathbf{x}_i) \approx \int_{\Omega_i} \nabla U(\mathbf{x})\widehat{w}(\mathbf{x}-\mathbf{x}_i)d\mathbf{x} \tag{2.1}$$

With piecewise constant smoothing function, the Eq. (2.1) can be simplified as

$$\nabla U_i \approx \frac{1}{V_i}\oint_{\partial\Omega_i} U\vec{n}ds \tag{2.2}$$

Eq. (2.2) gives an approximation of gradients at a point using a local smoothing domain, as implemented in the GSM.

As shown in Fig. 1, three types of gradient smoothing domains, which are used for the approximation of spatial derivatives, are constituted on primitive unstructured triangular cells. The first type of smoothing domain is the node-associated GSD (nGSD) for the approximation of derivatives at a node of interest. It is formed by connecting relevant centroids of triangles with midpoints of relevant cell-edges. The second is formed by a primitive cell, which is used for approximating derivatives at the centroid of the cell, as in cell-centred FVM. We call it centroid-associated GSD (cGSD) here. The third is

named midpoint-associated GSD (mGSD) used for the calculation of the gradients at the midpoint of a cell-edge of interest. The preferred mGSD is formed by connecting the end-nodes of the cell-edge with the centroids on the both sides of the cell-edge.

The overall adaptive process based on advancing front technique is shown in Fig. 2. It is proved that such a process is stable and efficient for solutions to both Poisson and Euler equations. The adaptive solutions to inviscid compressible flow over the NACA0012 airfoil are shown in Figs. 3 and 4.

Throughout the study, it is found that the proposed the proposed adaptive GSM is very stable, which provides more accurate solutions, thanks to the 'optimal' mesh achieved with the equidistribution criterion.

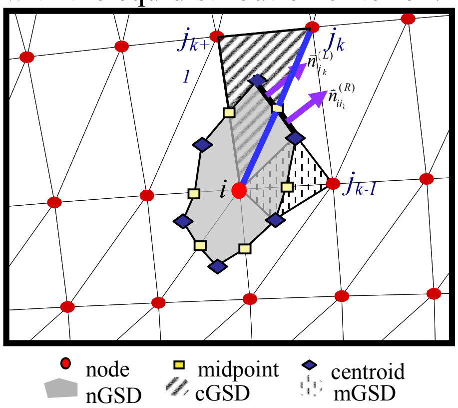

Fig.1 Gradient smoothing domains adopted in GSM

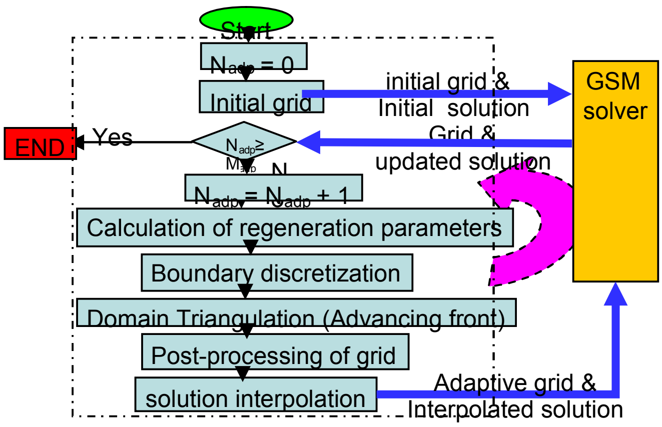

Fig.2 Adaptive process based on advanced front technique

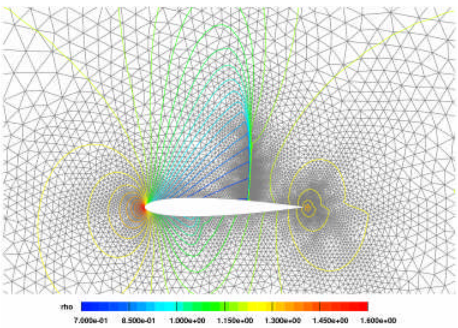

Fig. 3 Plots of adaptive meshes and corresponding contours of density

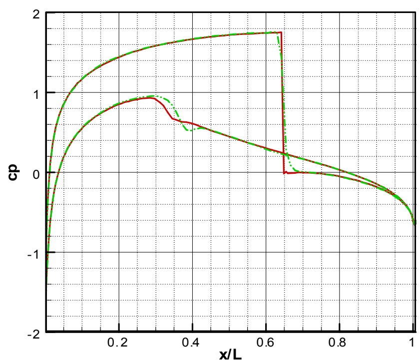

Fig. 4 Profiles of pressure coefficients on the NACA0012 airfoil surface

REFERENCES

[1] G. XU and G.R. Liu, Development of Irregular-Mesh Finite Difference Method (IFDM) for Governing Equations in Strong Form, Proceedings of the 4th WSEAS International Conference on Fluid Mechanics and Aerodynamics, pp. 154-159, Elounda, Greece, August 21-23, 2006.

[2] G.R. Liu and George X.G. XU, A Gradient Smoothing Method (GSM) For Fluid Dynamics Problems. Submitted to the International Journal for Numerical Methods in Fluid, 2007

[3] K. Morgan, J. Peraire and J. Peiro, Unstructured Grid Methods For Compressible Flows, AGARD-R-787, 1992.