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ACCURATE CONTINUOUS/DISCONTINUOUS VELOCITY APPROXIMATIONS FOR TRANSPORT IN POROUS MEDIA

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ABSTRACT

The flow of an incompressible homogeneous fluid in a rigid saturated porous media is usually modeled by the system of partial differential equations composed of the mass conservation equation plus Darcy's law that relates the average velocity of the fluid between the grains of the medium with a potential field, through the hydraulic conductivity tensor. This system is known as Darcy's problem.

Finite element approximations of this problem are essentially based on two different approaches: single-field formulation for potential [1,13] or mixed formulation in potential and velocity [2]. The main characteristic of the mixed finite element methods is the use of different approximating spaces for velocity and potential, requiring a compatibility condition between the finite element spaces to ensure existence and uniqueness of solution (LBB condition [3]) which reduces the flexibility in the choice of stable finite element spaces. One successful approach is the dual mixed one, developed by Raviart and Thomas [4], which uses divergence-free finite element spaces to approach the velocity field and discontinuous interpolation to compute the potential. This discontinuous evaluation of potential achieves mass conservation locally that is a celebrated property in transport applications. The use of these different discrete spaces is considered a drawback. Alternatively, stabilized finite element methods have been developed by adding residuals to the classical variational formulations to overcome this difficulty [5-11].

Stabilized formulations that use continuous Lagrangian finite element spaces for velocity have been successfully employed in simulating Darcy's flows in homogeneous porous media [5,6,9,11]. The point is their applicability to problems where the porous formation is composed by sub domains with different conductivities. On the interface between these sub domains, the normal component of Darcy velocity must be continuous (mass conservation) but the tangential component is discontinuous, and formulations based on continuous Lagrangian interpolation for velocity fails in representing the tangential discontinuity, producing inaccurate approximations and spurious oscillations [12,13]. On the other hand, stabilized formulations that use completely discontinuous approximations for velocity do not necessarily fit the continuity of the flux [7,8,12].

In this work we comment on some stabilized formulations presented in the literature for Darcy's problem and investigate the influence of the accuracy of the approximated velocity fields on the flow of

miscible species in saturated porous media. In the case of layered heterogeneous porous media with smooth interfaces of discontinuity of the hydraulic conductivity, we examine the use of continuous Lagrangian elements for velocity by adopting the additional stabilization on the interface proposed in [13], that incorporates the discontinuity on the tangential component of the velocity field in a strong sense.

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