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A STUDY ON SOLID CONTACT PROBLEM USING NODELESS METHOD BASED ON THE UNIFIED ENERGY PRINCIPLE

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ABSTRACT

In 1994, one of the authors derived the following new mixed variational principle for the state vector (u_i, σ_{ij}) of a solid from the energy conservation law using Gauss's divergence theorem[1)2)].

$$\delta \int_v \sigma_{ij}\varepsilon_{ij} dV - \int_v \bar{p}_i \delta u_i dV - \int_{S_\sigma} \bar{t}_i \delta u_i dS - \int_{S_u} \bar{u}_i \delta t_i dS = 0 \qquad (1)$$

Where $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ and $\sigma_{ij} = \sigma_{ji}$ are assumed, and $\bar{p}_i$ is body force in V, $\bar{u}_i$ is enforced displacement on S_u, $\bar{t}_i$ is surface traction on S_σ. Eq(1) is valid for deformation solids under isothermal or adiabatic temperature conditions.

An alternative form of above equation can be given by

$$\int_{S_\sigma} (t_i - \bar{t}_i)\delta u_i dS + \int_{S_u} (u_i - \bar{u}_i)\delta t_i dS - \int_V \left(\sigma_{ij,j} + \bar{p}_i\right)\delta u_i dV - \int_V u_i \delta\sigma_{ij,j} dV = 0 \qquad (2)$$

The last integral term of Eq(2) is important to study multi-physics problems. In case of pure mechanics problems, the last term can be deleted and eight different methods of solution can be proposed. Among them the well known the variational equation of Treffzs's method is given by

$$\int_{S_\sigma} (t_i - \bar{t}_i)\delta u_i dS + \int_{S_u} (u_i - \bar{u}_i)\delta t_i dS = 0 \quad \text{where} \quad \sigma_{ij,j} + \bar{p}_i = 0 \text{ in } V \qquad (3)$$

Displacement function of the 2D finite element is usuall defined by the following series

$$\begin{aligned} u &= a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 x^3 + a_8 x_2 y + a_9 xy^2 + a_{10} y^3 + \cdots \\ v &= b_1 + b_2 x + b_3 y + b_4 x^2 + b_5 xy + b_6 y^2 + b_7 x^3 + b_8 x_2 y + b_9 xy^2 + b_{10} y^3 + \cdots \end{aligned} \qquad (4)$$

Nodeless solution is got by solving simultaneous equation for undetermined coefficients under minimizing correlation potential between adjoining element using linear stress and strain relation. [3)4)]

In this study, the Trefftz's method using fourth order displacement function is used for solid contact problem. As definition of contact boundary, polygonal line approximation is adopted. An increment of compression load is forced on the line which links the reference point of both solids, and carry out a calculation to be based on the unified energy principle every each iteration.

As the 1st example, Hertz's 2D solution of the oval contact stress distribution is examined using the present nodeless method and results of this study is shown in Fig.1. In this problem the symmetrical elastic deformation are assumed both in punch(a cylindrical type) and work. In a result, the validity of the calculation solution by the nodeless method is confirmed.

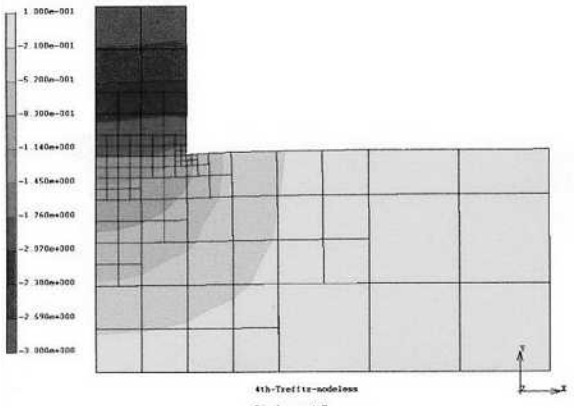

Fig.1 Result of Cylinder contact

As 2nd example, the cylindrical contact problem is executed comparison with the solution by the Trefftz's method and a conventional FEM technique. A punch and the transformation member considered symmetricalness. Enforced displacement is set for the line which is parallel in the x-axis in the Trefftz's method and is set each nodal point in FEM. The result of the present method is in good agreement with FEM.

As a results of nodeless method for solid contact problem with the Trefftz's method based on the unified energy principle using the 4th order displacement function, a proper result was provided and showed nonmatching element mesh that is a characteristic of the nodeless method. And present method showed possibility of solution contact problem. In near future more example of verification study on the more practical cases will be reported including frictional inelastic and dynamic contact.

REFERENCES

[1] T. Kawai, "Development of a Nodeless and Consistent Finite Element Method - force method forever-" , IACM WCCM Ⅴ, July 2002

[2] T. Kawai, "My Challenge in the Development of a Mixed Variational Method in Solid Mechanics", Applied Mechanics Reviews, Transaction of the ASME, Vol.60/P51-64, March 2007

[3] A. Kikuchi and E. Kazama, "Numerical examples of hybrid displacement method based on the unified energy principle", the Japan Society for Computational Engineering and Science Vol.12, May 2007

[4] A. Kikuchi and E.Kazama, " Numerical examples of nodeless finite element methods based on the unified energy principle", The Japan Society for Computational Engineering and Science Vol.12, May 2007