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An hp-adaptive Spacetime Discontinuous Galerkin Method with Discontinuity Tracking

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ABSTRACT

In previous work we proposed h-adaptive spacetime discontinuous Galerkin (SDG) methods for capturing shocks and other sharp solution features in hyperbolic problems [1–3]. This approach has a number of attractive features, including $\mathcal{O}(N)$ complexity, machine-precision balance properties on each spacetime element, support for higher-order bases on a fixed stencil, zero-projection error due to adaptive meshing, and a natural asynchronous parallel structure that facilitates scalable high-performance implementations. We use an h-adaptive version of the *Tent Pitcher* algorithm [4] to generate fully unstructured spacetime grids that adapt simultaneously in space and time. This circumvents the global time-step-size constraint that limits solution efficiency in conventional time-marching schemes. We exploit the flexibility of unstructured spacetime meshing to improve efficiency by concentrating refinement along the trajectories of shocks and other sharp solution features, as seen in Figure 1.

The h-adaptive SDG method is competitive with alternative finite element methods; it excels in problems requiring highly dynamic mesh adaptation, such as elastodynamic fracture. However, despite its superior adaptive remeshing capabilities, it often underperforms available finite difference and finite volume algorithms, especially when implemented with higher-order bases and applied to problems with irregular solutions. One cause of the problem is clear: as our h-adaptive scheme concentrates refinement along the trajectories of singular solution features, it increases the fraction of elements where solution regularity limits performance and the use of a higher-order basis delivers no benefit to mitigate its cost. Here we report two improvements to our adaptive algorithms that substantially improve computational efficiency: hp-adaptivity and discontinuity tracking.

We first describe an hp-adaptive SDG algorithm that inherits all the advantages of our h-adaptive scheme. We seek spacetime discretizations that use higher-order bases in regions where the solution is regular and lower polynomial orders in areas where the solution exhibits limited regularity or where mesh coarsening is inhibited by geometric constraints. A discontinuity indicator detects when a non-smooth, low-regularity solution feature traverses a spacetime element, triggering a local reduction in polynomial order; mesh refinement provides the required accuracy in these low-regularity regions. We also reduce the polynomial order in elements with overly precise solutions when geometric constraints

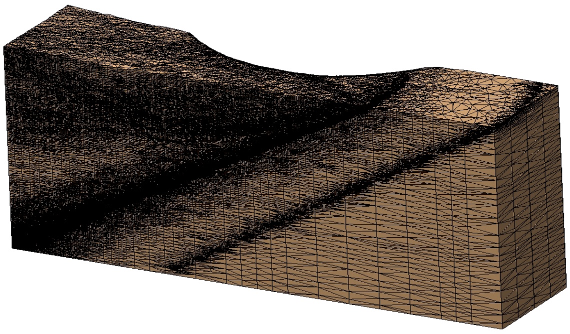

Figure 1: Spacetime-mesh showing intense h-adaptive refinement along trajectories of irregular solution features; time axis is vertical.

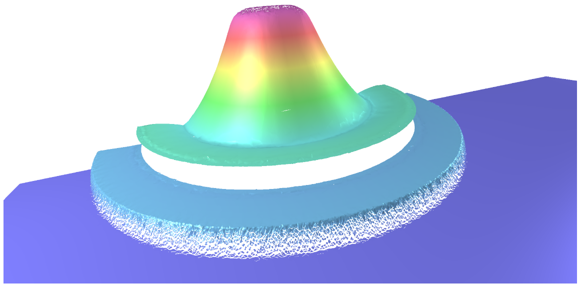

Figure 2: Tracking the contact discontinuity (inner jump) in cylindrical explosion example; shock (outer jump) is captured

prevent coarsening. Polynomial enrichment is preferred over mesh refinement in smooth regions where the adaptive error indicator calls for more resolution. As in the h-adaptive algorithm, the patch-by-patch, advancing-front SDG solution scheme supports highly dynamic adaptive enrichment/refinement. The hp-adaptive scheme delivers a substantial improvement in performance over its h-adaptive predecessor.

The second improvement is a new form of discontinuity tracking. We achieve improved discrete representations of the BV solution spaces for nonlinear conservation laws by constructing spacetime meshes where element faces cover the trajectories of singular surfaces. That is, jumps are *built into* our discrete solution space where they are needed. This admits more accurate approximations and dramatically reduces the problem size by eliminating the need to capture discontinuous features with intense mesh refinement. We use inclined 'tent poles' in an extended Tent Pitcher algorithm to track singular surfaces as the solution evolves. Spacetime versions of vertex-smoothing and edge-flip operations maintain the integrity and quality of the mesh. In contrast to previous attempts [3], here we do not assume *a priori* knowledge of the discontinuity's trajectory or that it can be represented exactly by a polyhedral surface. As seen in Figure 2, this approach delivers sharp renderings of discontinuities, while dramatically reducing solution cost relative to traditional capturing techniques. Tracking can be combined with hp-adaptivity to achieve even greater computational savings.

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