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AN ALE BASED CBS ALGORITHM FOR NON-ISOTHERMAL NON-NEWTONIAN FLOW WITH ADAPTIVE COUPLED FINITE ELEMENT AND MESHFREE METHOD

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ABSTRACT

This paper derives a new version of the characteristic Galerkin method [1-3] in the ALE discription, in which temporal discretization for material time derivative of a physical variable φ at a given reference particle is devised in a general implicit form and $\varphi(\mathbf{X}(t_{n+1}), t_{n+1})$ and $\varphi(\mathbf{X}(t_n), t_n)$ for the reference particle are approximated in the form

$$\varphi(\mathbf{X}(t_{n+1}), t_{n+1}) = \varphi^{n+1} + (1-\alpha)\frac{\Delta t}{2}\left[(1+\alpha)\mathbf{c}^{n+1} + (1-\alpha)\mathbf{c}^{n}\right]\cdot\nabla\varphi^{n+1} + (1-\alpha)^2\frac{\Delta t^2}{2}\mathbf{c}^{n+\beta}\cdot\nabla\left(\mathbf{c}^{n+\beta}\cdot\nabla\varphi^{n+\beta}\right) + O\left(\Delta t^3\right) \quad (1)$$

$$\varphi(\mathbf{X}(t_n), t_n) = \varphi^{n} - \alpha\frac{\Delta t}{2}\left[\alpha\mathbf{c}^{n+1} + (2-\alpha)\mathbf{c}^{n}\right]\cdot\nabla\varphi^{n} + \alpha^2\frac{\Delta t^2}{2}\mathbf{c}^{n}\cdot\nabla\left(\mathbf{c}^{n}\cdot\nabla\varphi^{n}\right) + O\left(\Delta t^3\right) \quad (2)$$

where any values within $0 \le \alpha \le 1$ and $0 < \beta \le 1$ can be taken, $\mathbf{c}$ the convective velocity defined as $\mathbf{c} = \mathbf{u} - \hat{\mathbf{u}}$ with mesh velocity.
Then a generalized scheme of iterative stabilized fractional step algorithm (FSA) in ALE framework using Characteristic Based Split (CBS) for the numerical simulation of incompressible non-isothermal non-Newtonian fluid flows is developed. Further studies of the FSA[4] concluded that it can circumvent the restriction imposed by the LBB condition only if non-incremental version of the algorithm is used with the time step size larger than a critical value. To enhance the numerical stability of the FSA particularly to modeling of high or moderate viscosity fluid flow and to allow the use of the incremental version of the FSA, finite element calculus (FIC) process[5] is introduced in this work [6] to re-form the FIC stabilized mass conservation equation.

To further enhance the incompressibility stability of the proposed PS_FSA, an iterative version I_PS_FSA was proposed. It is based on introducing an iterative procedure into the algorithm to make both diffusive and convective terms satisfy the momentum conservation equation in an implicit sense, which allows much larger time step sizes to be used for the numerical solutions of incompressible N-S equations with different

values of the Reynolds number ranging from low to high viscosities and reduces in a decisive manner the computational effort[7].

To eliminate the instability of numerical solutions due to the hyperbolic nature of the convection operator in incompressible N-S equations, characteristic-based split (CBS) is introduced into the proposed I_PS_FSA to form a generalized scheme of the FSA using CBS abbreviated as I_PS_CBS in ALE framework for numerical simulation of incompressible non-isothermal non-Newtonian fluid flows. The coupling of energy conservation equation with the incompressible N-S equations are taken into account and temperature, in addition to $\mathbf{u} - p$, is involved as one of primary variables[8].

To further enhance both accuracy and efficiency of the simulation, an adaptive coupled FE and Meshfree method (MF) in ALE description, in which the respective strong points of the FE and MF methods are adequately exploited while their respective weak pints are effectively suppressed, developed in [9] is extended to simulate non-isothermal non-Newtonian flow with moving free surfaces injection molding processes and particularly applied to the polymer injection molding process.

REFERENCES

[1] Zienkiewicz OC and Codina R, “A general algorithm for compressible and incompressible flow-Part I. The split, characteristic based scheme”, Int. J. Numer. Methods Fluids, Vol. **20**, pp. 869-885 (1995).

[2] Codina R, Numerical solution of the incompressible Navier-Stokes equations with Coriolis forces based on the discretization of the total time derivative, Journal of Computational Physics, Vol. **148**, pp. 467-496 (1999).

[3] Li XK, Wu WH and Zienkiewicz OC, Implicit characteristic Galerkin method for convection - diffusion equations, Int. J. for Numerical Methods in Eng, Vol. **47**, pp.1689-1708 (2000).

[4] Guermond JL and Quartapelle L, On stability and convergence of projection methods based on pressure Poisson equation, Int J Numer Methods Fluid, Vol. **26**, pp.1039-1053 (1998).

[5] Oñate E. A stabilized finite element method for incompressible viscous flows using a finite increment calculus formulation, Comput. Methods Appl. Mech. Engrg. Vol. **182**, pp. 355-370 (2004).

[6] Li XK and Duan QL, Meshfree iterative stabilizaed Taylor-Galerkin and characteristic- based split (CBS) algorithms for incompressible N-S equations, Comput. Methods Appl. Mech. Engrg., Vol. **195**, pp.6125-6145 (2006).

[7] Li XK and Han XH, An iterative stabilized fractional step algorithm for numerical solution of incompressible N-S equations, Int. J. for Numerical Methods in Fluids, Vol. **49**, pp. 395-416 (2005).

[8] Han XH and Li XK, An iterative stabilized CNBS-CG scheme for incompressible non-isothermal non-Newtonian fluid flow, Int. J. Heat Mass Transfer, Vol.**50**, pp.847-856 (2000).

[9] Li XK, Duan QL, Han XH and Sheng DC, Adaptive coupled arbitrary Lagrangian-Eulerian finite element and meshfree method for injection molding process, Int. J. Numerical Methods in Eng, DOI:10.1002/nme.2117, 2008.