8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

SPACETIME METHOD FOR TRACKING ELASTODYNAMIC FRACTURE WITH A DAMAGE-BASED COHESIVE MODEL

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Key Words: *Dynamic Fracture, Discontinuous Galerkin, Damage, Cohesive Model, Adaptive Analysis.*

ABSTRACT

In previous work [1,2,3], we proposed a spacetime discontinuous Galerkin (SDG) model for elastodynamic fracture. Our h-adaptive implementation was able to automatically resolve the fine detail of the dynamic fracture process zone, but its application was restricted to problems where the potential crack trajectories could be defined *a priori*. In the work reported here, we propose a new method wherein an expanded set of adaptive spacetime meshing operations supports predictions of crack nucleation, propagation and branching along trajectories that are determined during the solution process. We also introduce a new damage-based cohesive model that models a smooth transition from an initially rigid interface to one governed by a cohesive traction condition.

Our method inherits the favarable properties of the SDG method for elastodynamics [1]. It uses basis functions defined on fully unstructured spacetime meshes to describe displacement solutions that admit jumps across all inter-element boundaries. This discontinuous solution structure leads to exact balance of linear and angular momentum on every spacetime element and superior shock-capturing properties. When implemented on suitable spacetime grids, the SDG method exhibits linear complexity in the number of elements. The SDG formulation easily incorporates cohesive damage models; displacement jumps are intrinsic to the model, so the only modification is the use of the cohesive traction model to define the target momentum flux on cohesive interfaces. There is no need for cohesive elements or other special methods.

Our SDG implementation includes an h-adaptive spacetime meshing procedure [4] that ensures accurate resolution of sharp wavefronts and sufficient refinement in the active fracture process zone to ensure numerical stability and an accurate rendering of the TSL. Two independent error indicators drive the adaptive procedure: a dissipation-based indicator that limits numerical energy dissipation throughout the solution domain and one that controls the discrepancy between the works of separation predicted by the trace of the finite element stress field and the cohesive traction model. The resulting high-precision solutions led to the discovery of quasi-singular velocity response in the neighborhood of the process zone and the first transient studies of the nonlinear relation between crack velocity and process-zone size.

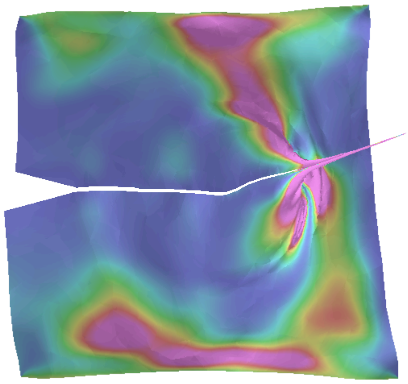

Figure 1: Dynamic fracture under mixed-mode loading showing quasi-singular velocity field

In new work reported here, we extend the adaptive meshing capabilities to support solution-dependent nucleation, extension and branching of cohesive surfaces. We introduce a new set of spacetime adaptive meshing operations where each operation is implemented as a special spacetime patch rather than as a discrete operation in space. The inflow faces of the special patches conform to the outflow faces of previously solved elements, so there is no need to project the old solution onto a new mesh. This eliminates the projection errors incurred by conventional adaptive remeshing procedures and preserves the full convergence rates of high-order elements. Patches with *inclined tent poles* reposition vertices in the space mesh; we use these to continuously smooth the space mesh to maintain and improve its quality and to track moving interfaces. Special single-tetrahedron patches perform *edge-flip operations* improve the quality of the spatial triangulation. *Coarsening patches* remove a vertex from the space mesh. Mesh refinement involves a nested subdivision of the space mesh that incurs zero projection error. We use these operations in combination to nucleate cohesive interfaces at arbitrary locations and to extend existing interfaces in any direction, as indicated by the physics of the solution. Element quality is maintained throughout the procedure, and there are no restrictions on the direction of crack propagation. Figure 1 shows an example of dynamic crack propagation under mixed-mode loading conditions.

Initially-rigid models are commonly used in cohesive models of dynamic fracture, especially with explicit time integration schemes. These models transition abruptly from an undamaged state to the regime of a traction-separation law, and are, therefore, non-differentiable. This makes them unsuitable for our patch-wise-implicit SDG solution scheme. We propose a new class of initially-rigid cohesive models that use a damage parameter to transition smoothly between enforcing the flux conditions for undamaged material and the traction conditions defined by the cohesive model. The damage parameter can be interpreted as the local area fraction of the cohesive interface that is active in the separation process.

Acknowledgment: This work was supported by the Center for Process Simulation & Design, University of Illinois at Urbana Champaign under U.S. National Science Foundation grant DMR-01-21695.

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