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A STUDY ON OPTIMAL STRENGTH DISTRIBUTION OF RIGID-PLASTIC BODY

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ABSTRACT

Designing civil engineering structures with plasticity in mind has become an important research topic.

If elastic deformation can be ignored, structures may be viewed as a rigid plastic body. This enables an analysis which does not require a step-by-step calculation like that of elasto-plastic bodies. In the case of a complicated problem like an optimal design problem, the incremental step-by-step analysis needs much computational work. In this case though, the optimal design of a rigid plastic body problem can avoid lengthy calculations and the solution can be obtained by simple formulation. A method for the optimum design of a rigid plastic body has been proposed by Kaliszky et al.[1]. However, their study was limited to simple structure, such as bars and disks. In the present study, we propose the method for the optimum design of “a rigid plastic continuum”. By using this method, an optimum design against ultimate limit state can be obtained.

In this paper, “optimum” is defined as the lowest cost of construction, that is, an object function of the optimum problem is cost (cost function). A variable of the cost function is a material parameter which is a measure of strength against plastic yield. This parameter henceforth is referred as simply “strength”, which includes apparent cohesion of soil or full plastic moment of beams. Next, constraint conditions are “equilibrium of forces”, “condition of yield function”, and “condition of strength”. The first two are “static conditions” of the limit analysis, while the third condition is that value of strength must have lower bound, upper bound, or both of them.

For convex programming problems or problems that can be approximated to a convex programming problem, Lagrange’s method of undetermined multipliers can be utilized to solve the problem. A matrix of the system of equations, that is to be solved, can become a positive-definite symmetric band matrix by using a Quasi-Newton method[2] such as BFGS. Therefore, the present method requires little computational work.

As an optimum design example, consider the optimum distribution design of a bearing capacity problem in Figure 1. This problem simulates how to reinforce the soft ground. Here, a design variable is the

apparent cohesion of ground c. Cohesion c is assumed to be not smaller than its original value c_0. This means that cohesion c increases from c_0 by reinforcement. An objective function is defined as total increment of cohesion ($\int_V (c - c_0)\mathrm{d}V$). Figure 1 shows a finite element mesh and loading conditions. Figure 2 show a result of a usual rigid plastic analysis[3] for homogeneous ground ($c = c_0$ in V). Here, deep black indicates large value of deformation and white indicates no deformation respectively. In the case of homogeneous ground (not reinforced ground), a limit distribution load $q = (\pi + 2)c_0$ is obtained analytically. Figure 3 shows a result of an optimum distribution of cohesion. In this problem, distribution load is designate to be $q = 2(\pi + 2)c_0$, which is just twice as large as a limit load for homogeneous ground. Deep black indicates large value of reinforced cohesion c, while white indicates c_0. Figure 4 shows a result of a rigid plastic analysis for the optimized ground shown in figure 3. As shown in Figures 4, a slip surface is larger than that in Figure 2. The largely extended slip surface exhibits more friction force which sustains the distributed load.

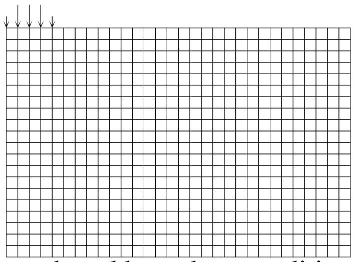

Fig. 1. mesh and boundary conditions using analysis

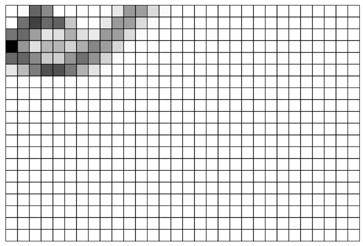

Fig. 2. distribution of equivalent strain rate at plastic collapse (homogeneous ground)

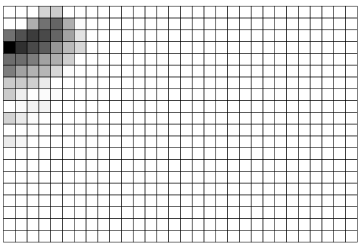

Fig. 3. distribution of “cohesion” (optimal distribution)

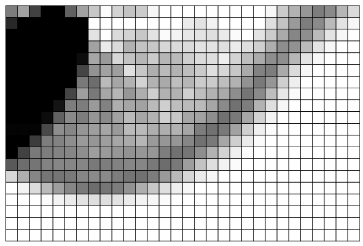

Fig. 4. distribution of equivalent strain rate at plastic collapse (optimal distribution)

The conclusions are as follows.

- The validity of the formulation and programming has been shown by a analysis of a cantilever. (An optimum design problem of a cantilever is not provided in this abstract due to the restriction of the paper.)
- It has been shown by the analysis of bearing capacity that the method proposed in this study could be applied to the rigid plastic continuum.

The presented method can be extended to optimal shakedown design of an elasto-plastic continuum in the same way that Kaliszky et al.[1] Although this paper deals with a civil engineering problem, the method can be applied to problems in the field of geosciences.

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