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Quantification of Uncertainties in Computational Mechanics Simulations Using Experimental Uncertainty Analysis Concepts

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ABSTRACT

A rational treatment for modeling and quantifying uncertainties in computational mechanics simulations using concepts from experimental uncertainty analysis is discussed. The approach is that used in the new ASME V&V 20: Standard for Verification and Validation in Computational Fluid Dynamics and Heat Transfer [1], and the concepts and techniques are equally applicable in other fields of computational mechanics. The estimation of a range within which the simulation modeling error lies is a primary objective of the validation process. This is accomplished for a specified validation variable at a specified set of conditions by comparing the simulation result (solution, S) with an appropriate experimental result (data, D) and considering the errors and uncertainties associated with both S and D [2-5].

Previously published AIAA and ASME V&V Guides [6, 7] present the philosophy and procedures for establishing a comprehensive validation program, but use definitions of error and uncertainty that are not demonstrated within the guides to provide quantitative evaluations of the comparison of the validation variables predicted by simulation and determined by experiment. Reference 7, for instance, defines error as "a recognizable deficiency in any phase or activity of modeling or experimentation that is not due to lack of knowledge" and defines uncertainty as "a potential deficiency in any phase or activity of the modeling, computation, or experimentation process that is due to inherent variability or lack of knowledge."

In contrast, the V&V 20 approach is based on the concepts and definitions of error and uncertainty [2-5] that have been internationally codified by the experimental community over several decades. These concepts are applied to the errors and uncertainties in the experimental result D and also to the errors and uncertainties in the result S from the simulation. The error in the experimental result D is δ_D, and errors in the simulation result S are: δ_{model} due to modeling assumptions and approximations; δ_{num} due to the numerical solution of the equations; and δ_{input} due to errors in the simulation input parameters. Following the ISO Guide [2], for each error source (other than the simulation modeling error) a standard uncertainty, u, is estimated such that u is the standard deviation of the parent population of possible errors from which the current error is a single realization. This allows estimation of a range within which the

simulation modeling error lies.

The validation metrics used are the validation comparison error $E = S - D$ and the validation uncertainty u_{val}, which is the standard uncertainty that characterizes an interval which includes the combination of errors ($\delta_{num} + \delta_{input} - \delta_D$). The validation uncertainty u_{val} is composed of contributions from the standard uncertainties u_{num}, u_{input}, and u_D. The uncertainty u_{num} is estimated as a result of code and solution verification procedures [8, 9]. The contribution of the combination of u_{input} and u_D is determined by propagation of simulation input uncertainties and experimental uncertainties using either a sensitivity coefficient approach [2] or a Monte Carlo (sampling) approach [3] and taking into account the correlation effects of shared variables in S and D and multiple measured variables possibly sharing identical elemental error sources.

Examples of application of the approach will be discussed for a case in which the validation variable D is directly measured (and thus S and D have no shared variables) and a case in which D is determined from a data reduction equation that combines multiple measured variables (and thus S and D have shared variables and a correlation effect must be considered.)

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