

**NEW AND INNOVATIVE ADVANCES AND STATE-OF-THE ART
CONCEPTS IN TIME INTEGRATION: A CONSISTENT DESIGN
METHODOLOGY FOR STRUCTURE
PRESERVING/CONSERVING VERSUS NUMERICAL
DISSIPATIVE INTEGRATORS FOR COMPUTATIONAL
FLEXIBLE/RIGID DYNAMICS**

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ABSTRACT

In this invited Keynote Lecture on Computational Methods in Multi-Body Dynamics Simulation (ID 19), new and recent advances and innovative concepts that are state-of-the-art and which set forth future directions towards the appropriate design of time integration of the equations of motion arising in computational dynamics are described under the simple notion of *Algorithms By Conceptual Design (ABCD)*. In contrast to past approaches and classical paradigms, the theoretical developments leading naturally via a consistent mathematical setting (without resorting to added enforcement of constraints such as energy, momentum, and the like *a posteriori*, to achieve desired algorithmic attributes) to enable new designs of conserving time operators (attributes such as energy, linear and angular momentum conservation) are first described for nonlinear/linear computational dynamics applications. This is next followed by extensions of these basic parent conserving time operators to now include aspects of controllable numerical dissipation features in the designs with optimal algorithmic attributes necessary to preserve as much physics as possible [1]. Obviously, the inclusion of controllable numerical numerical dissipation indeed destroys some of the conservation aspects [2]; however, for several practical applications, they help foster the total simulation time window over their conservative counterparts. And, naturally when the numerical dissipative features are turned off, they revert back to the parent conserving operators. The applications of the present designs are of fundamental interest to the broad research community at large dealing with general linear or nonlinear dynamics of flexible and/or rigid structural configurations, and of immense benefit to code developers in commercial software. Design, modeling, and simulation assessments of both conserving time operators versus the inclusion of controllable numerical dissipative time integrators are described to elucidate the overall developments, including plausible strategies of intermittent *Hand Shaking* concepts within the same analysis simulation to enable a long term simulation time window. For purposes of illustration, all of these developments are described in the context of and with focus on Linear Multi-Step (LMS) methods which are the most practical

and popular in commercial and research software for general applications [3].

Considering the necessity to appropriately integrate the equations of motion arising in general dynamics applications for rigid, flexible or both bodies coexisting in the same structural configuration, for computational algorithms there is in general, a division of time integrators as options, such as energy and momentum conserving or the so called symplectic and momentum conserving for unconstrained dynamic systems. There have been different schools of thought in the respective designs for these two classes of algorithmic structures.

In one school of thought, one integrates the conservation form of the equations of motion in the context of the Hamiltonian or Newtonian mechanics description and then employs constraints such as energy and the like *a posteriori*, to foster the design of the so called energy-momentum type method to preserve algorithmic properties such as energy and linear and angular momentum conservation. In an attempt to enable the solution of stiff systems, strategies to introduce controllable numerical dissipation have also been additionally made since the non-dissipative algorithms faced convergence issues due to their inability to handle nonlinear stability although such numerical dissipative aspects are well known to destroy some of the conservation properties.

Alternately, one employs the so called Discrete Euler Lagrange Formulations in the context of Lagrangian Mechanics description to enable the design of structure preserving or symplectic momentum preserving algorithms. It is to be also noted that literature cites that both the classes cannot co-exist with constant time step algorithms and this is the price one has to pay. Also, while strict energy conservation with arbitrary time step value appears to be attainable via energy momentum approaches thus alleviating the issues regarding nonlinear stability, it appears to be not attainable via the symplectic momentum approaches; rather, the energy is indeed only bounded and that too for only small enough time steps.

In the present lecture, we describe a consistent mathematical setting and computational framework with focus on the Saint Venant Kirchoff material and Green strain for illustration, to describe new avenues and developments and new designs that naturally and inherently lead to the various classes of algorithms described earlier which enable conserving/preserving attributes. This is naturally attained without the need to additionally impose any artifacts or any constraints such as energy and the like *a posteriori*, and/or resort to different mechanics descriptions to foster such designs. Furthermore, when necessary, controllable numerical dissipative features with optimal design attributes can be easily included to handle a wide class of applications. Their structure and design is such that when the numerical dissipative features are turned off, they naturally revert to conserving/preserving algorithms designed previously. Several numerical illustrations demonstrate the overall concepts for engineering applications.

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