8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 - July 5, 2008
Venice, Italy

Inverse Point Load Superposition Thermoelastic-BEM Cavity Detection Technique

E. Divo[1], A.J. Kassab[2], D. Ojeda[3], M. Cerrolaza[4]

[1]Engineering Technology Department
University of Central Florida
Orlando, Florida 32816, USA
edivo@mail.ucf.edu

[2]Mechanical, Materials and Aerospace Engineering
University of Central Florida
Orlando, Florida 32816, USA
kassab@mail.ucf.edu

[3]Mechanical Design and Automation, Engineering
University of Carabobo
Valencia, Venezuela
dojeda@uc.edu.ve

[4]National Institute of Bioengineering
Central University
Caracas, Venezuela
mcerrola123@gmail.com

Key Words: *BEM; Cavity detection; Genetic algorithm; Thermoelasticity.*

ABSTRACT

A method for the efficient solution of the inverse optimization problem of cavity detection using a point load superposition technique in thermoelastic boundary element methods is presented in this paper. The superposition of point load clusters in the domain is posed as an alternative to satisfy the Cauchy conditions on the surface. Using Genetics Algorithms, the point load distribution, strength, and location are altered to seek satisfaction of the over-specified boundary displacements. The superposition of point loads as an alternative to simulate the presence of a cavity offers tremendous computational advantages as it allows to pose the inverse problem as a search for the optimal strength and location of such point loads rather than requiring the remeshing of the sought for cavity and thus the recomputation of coefficients and full solution of algebraic systems at every step of the optimization problem.

The solution of the forward thermoelastic problem is expressed in terms of displacements which, for an isotropic, homogeneous, and linearly elastic medium imposed with an internal volumetric force b_i, is governed by Navier's equation as:

$$\mu \frac{\partial^2 u_i}{\partial x_i \partial x_j} + \left(\frac{\mu}{1-2\nu}\right)\frac{\partial^2 u_j}{\partial x_i \partial x_j} + b_i = m\frac{\partial T}{\partial x_i} \tag{1}$$

Here, $m = \frac{2\mu\alpha(1+\nu)}{(1-2\nu)}$, $\underset{\sim}{u}$ is the displacement vector, ν is Poisson's ratio, μ is shear modulus, and α is the coefficient of thermal expansion. Introducing the fundamental solution to Navier's equation, a BEM formulation can be derived from the Somigliana identity providing an integral relation between the displacement vector u_i^p in a point collocation "p" as:

$$c_{ij}^p u_i^p + \oint_\Gamma H_{ij} u_i d\Gamma = \oint_\Gamma G_{ij} t_i d\Gamma + \frac{m}{k}\left[\oint_\Gamma E_j q d\Gamma - \oint_\Gamma F_j (T - T_{ref}) d\Gamma\right] + \sum_{l=1}^{NL} Q_i^l G_{ij}^l \tag{2}$$

Where the internal force b_i is formed only by NL points loads, so that $b_i = \sum\limits_{l=1}^{NL} Q_i^l \delta(x_i, x_i^l)$.

To form the preliminary algebraic system, the point p is collocated at all nodes NN of all the elements NE to generate equations of the form $[H]\{u\} = [G]\{t\} + \{s\} + \{q\}$. The vector $\{s\}$ contains the integrated information of the thermal effects on the elastic field and all effects generated by points loads are located in the vector $\{q\}$, therefore, when point loads that are utilized in the inverse geometric problem solution are relocated in the evolving solution, there is no need for boundary re-meshing as the approach proposed in this paper for the modeling of internal cavity(ies) is inspired from potential theory where the superposition of a source and a sink with the same strength located a distance L in a prescribed parallel flow results in iso-flow lines and simulate the presence of a solid surface through the iso-lines containing stagnation points, see Figure 1.

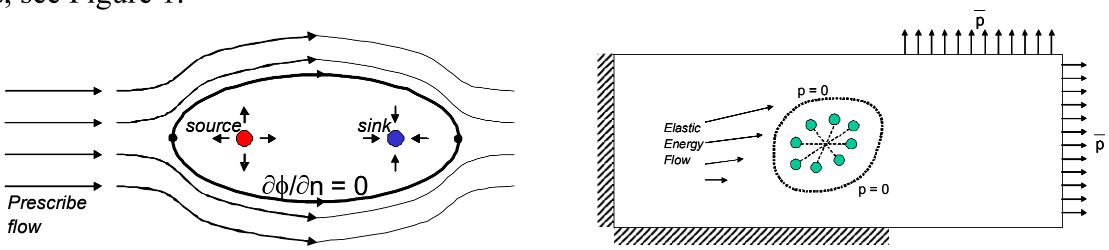

Figure 1. Simulation of surface with incoming flows by singularities superposition.

As such, the first Genetic Algorithm searches for locations and strengths of the fictitious point loads until a match is found between the tractions and deformations computed by the BEM and those measured on the boundary as additional information or over-specified conditions and the second Genetic Algorithm searches for the cavity(ies) location(s) indicated as traction-free surface(s) around the point loads. This is accomplished by defining two independent objective functions formulated as least-squares differences of these quantities.

An example considers a square section with a centered hole. The sought for cavity is located on the right-hand side as shown in Fig. 2. Compressive loads are applied on three surfaces and a linear temperature field is imposed in the horizontal direction. The displacement measurements are simulated by applying random error to the direct BEM thermoelastic solution.

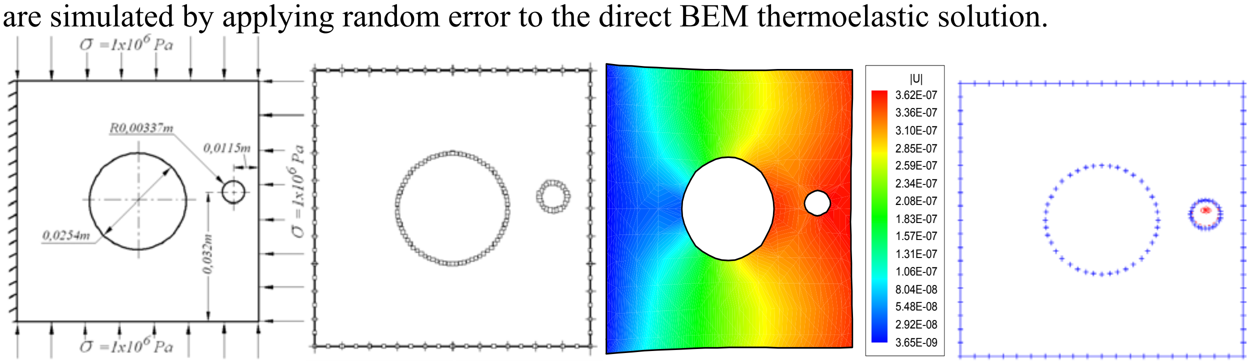

Figure 2. Geometry, conditions, discretization, direct solution, and cavity detection.

REFERENCES

[1] Ozisik N. M., Orlande H., *Inverse heat transfer,* Taylor&Francis, New York, 2001.

[2] Divo E.A., Kassab A.J., Rodríguez F., An efficient singular superposition technique for cavity detection and shape optimization. Numerical Heat Transfer, Vol. 46: 1-30, 2004.