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IDENTIFICATION OF DIFFUSIVITY COEFFICIENTS IN TIME FRACTIONAL DIFFUSION EQUATIONS

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ABSTRACT

The identification of parameters in classical parabolic equations is an ill-posed problem that has received considerable attention from many researchers in a variety of fields, using different methods. In particular, the determination of diffusivity coefficients in the one and two dimensional inverse heat conduction problem (IHCP) is a parameter identification type of question that has been extensively explored.
In this paper, if the temperature distribution is known approximately on the entire domain of interest, we extend the identification of diffusivity coefficients to transport processes with long memory where the rate of diffusion is inconsistent with the classical Brownian motion model.
We introduce a regularization technique for the approximate reconstruction of spatial, time and fractional order varying diffusivity coefficients using the observed solutions of the forward time fractional diffusion problem on a discrete set of points.
Basic properties and estimates associated with mollification and fractional derivatives are stated and fractional diffusion equations are introduced.
A fully implicit unconditionally stable numerical method needed to generate the data for the inverse problem when modeling is also described.
The numerical method is based on stable computations of Caputo partial time fractional derivatives and second order space partial derivatives by adaptive filtered versions of the noisy data using the method of discrete mollification.
The stabilized identification problem and the corresponding stability and error analysis of the approximate solution are provided.
To illustrate the efficiency of the proposed identification approach, a representative numerical example of interest is also presented and analyzed.

REFERENCES

[1] M. Caputo, *Elasticà e Dissipazione*, Zanichelli, Bologna, (1969)

[2] G. Chavent and J. Jaffre, *Mathematical Models and Finite Elements for Reservoir Simulation*, North Holland, Amsterdam, (1986)
[3] C. Coles and D. A. Murio, "Simultaneous space diffusivity and source term reconstruction in 2D IHCP", *Comp. Math. Applic.*, **42**, 1549-1564, (2001)

[4] R. Ewing, T. Lin and R. Falk, "Inverse and ill-posed problems in reservoir simulation", *Inverse and Ill-Posed Problems*, H. W. Engl and C. W. Groetsch, Eds., Academic Press, Orlando, 483-497, (1987)

[5] R. Ewing and T. Lin, "Parameter identification problems in single-phase and two-phase flow", *International Series of Numerical Mathematics*, **91**, Birkhauser Verlag, Basel, 85-108, (1989)

[6] C. E. Mejía, and D. A. Murio, "Numerical identification of diffusivity coefficient and initial condition by discrete mollification", *Comp. Math. Applic*., **30** (12), 35-50, (1995)

[7] D. A. Murio, "Implicit finite differences approximation for time fractional diffusion equations", to appear in *Comp. Math. Applic*., (2007)

[8] D. A. Murio, "On the stable computation of Caputo fractional derivatives", *Comp. Math. Applic.*, **51**, 1539-1550, (2006)

[9] D. A. Murio, "Mollification and Space Marching", Chapter 4 in *Inverse Engineering Handbook*, K. Woodbury Ed., CRC Press, Boca Raton, Florida, 219-326, (2002)

[10] I. Podlubny, *Fractional Differential Equations*, Academic Press, (1999)

[11] M. F. Wheeler, Editor, *Numerical Simulations in Oil Recovery*, Springer-Verlag, New York, (1988)